Corona Límites 2

1) $f(x) = \frac{\sqrt{-3x+6x^2}}{x^2+2}$

- $\lim_{x \to -\frac{1}{2}} f = 0,8$
- $\lim_{x \to \frac{1}{2}} f = \frac{0}{\frac{1}{4}+2} = 0$
- $\lim_{x \to 0} f = \left\{ \frac{0}{2} = 0 \right.$
- $\lim_{x \to +\infty} f = \frac{+\infty}{+\infty} = 0 \leftarrow \left(\frac{\text{grado } 1}{\text{grado } 2} \right)$
- $\lim_{x \to -\infty} f = \frac{+\infty}{+\infty} = 0$

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2) $f(x) = \frac{\sqrt{2x^4+5x^3-2x^2}}{3x-1}$

- $\lim_{x \to 0} f = \frac{0}{3} = 0$
- $\lim_{x \to +\infty} f = \frac{+\infty}{+\infty} = +\infty \leftarrow \left(\frac{\text{grado } 2}{\text{grado } 1} \right)$
- $\lim_{x \to -\infty} f = \frac{+\infty}{-\infty} = -\infty$

Corona Límites 2

3)
- $\lim_{x \to -4} f = \frac{2}{-4} = -\frac{1}{2}$
- $\lim_{x \to 4} f = \frac{2-\sqrt{8}}{4} = -0,2$
- $\lim_{x \to 0} f = \left(\frac{2-2}{0} \right) = \left(\frac{0}{0} \right) \to$ Simplificar Racionalizar

$$\frac{2-\sqrt{x+4}}{x} = \frac{(2-\sqrt{x+4})(2+\sqrt{x+4})}{x(2+\sqrt{x+4})} = \frac{2^2-\sqrt{x+4}^2}{x(2+\sqrt{x+4})} =$$

$$= \frac{4-x-4}{x(2+\sqrt{x+4})} = \frac{-x}{x(2+\sqrt{x+4})} = \frac{-1}{2+\sqrt{x+4}}$$

$$\lim_{x \to 0} \frac{-1}{2+\sqrt{x+4}} = \frac{-1}{2+\sqrt{4}} = \frac{-1}{4}$$

- $\lim_{x \to +\infty} f = \frac{-\infty}{+\infty} = 0 \leftarrow \left(\frac{\text{grado } \frac{1}{2}}{\text{grado } 1} \right)$
- $\lim_{x \to -\infty} f = \nexists$

Geogebra

Corona Límites 2

6) • $\lim\limits_{x \to -\frac{1}{2}} f = 3{,}5$

• $\lim\limits_{x \to \frac{1}{2}} f = 3{,}9$

• $\lim\limits_{x \to 0} f = 0 - (-3) = 3$

• $\lim\limits_{\substack{\cancel{x \to +\infty} \\ x \to -\infty}} f = \sqrt{+\infty} - (-\infty) = \sqrt{+\infty} + \infty =$
$= +\infty$

• $\lim\limits_{x \to +\infty} f = \sqrt{+\infty} - (+\infty)$ (grado 1 - grado 1)
Indeterminación
Racionalizar

$$\frac{\left[\sqrt{4x^2+2x} - (x-3)\right]\left[\sqrt{4x^2+2x} + (x-3)\right]}{\sqrt{4x^2+2x} + (x-3)} =$$

$$= \frac{\sqrt{4x^2+2x}^{\,2} - (x-3)^2}{\sqrt{4x^2+2x} + (x-3)} = \frac{4x^2+2x - (x^2-6x+9)}{\sqrt{4x^2+2x} + (x-3)} =$$

$$= \frac{3x^2+8x-9}{\sqrt{4x^2+2x} + (x-3)} \; ;$$

$$\lim_{x \to +\infty} f = \frac{+\infty}{+\infty} = +\infty$$

$\left(\frac{\text{grado } 2}{\text{grado } 1}\right)$

Geogebra

Corona Límites 2

(4) • $\lim_{x\to -2} f = \nexists$

• $\lim_{x\to 2} f = \left(\frac{0}{0}\right) \to$ Simplificar
Racionalizar

$$\frac{\sqrt{x}-\sqrt{2}}{x-2} = \frac{(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})}{(x-2)(\sqrt{x}+\sqrt{2})} = \frac{\sqrt{x}^2-\sqrt{2}^2}{(x-2)(\sqrt{x}+\sqrt{2})} =$$

$$= \frac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{x}+\sqrt{2})} = \frac{1}{\sqrt{x}+\sqrt{2}}$$

$$\lim_{x\to 2} \frac{1}{\sqrt{x}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}} = 0,4$$

• $\lim_{x\to 0} f = \frac{-\sqrt{2}}{-2} = 0,7$

• $\lim_{x\to +\infty} f = \frac{+\infty}{+\infty} = 0 \Leftarrow \left(\frac{\text{grado } \frac{1}{2}}{\text{grado } 1}\right)$

• $\lim_{x\to -\infty} f = \nexists$

Corona Límites 2

(5) • $\lim_{x\to \cancel{-\infty}\, -4} f = \frac{5}{\sqrt{0}} = \pm\infty$ (Laterales)

Geogebra

$\lim_{x\to -4^-} f = \frac{5}{\sqrt{-0}} = \nexists$

$\lim_{x\to -4^+} f = \frac{5}{\sqrt{+0}} = +\infty$

• $\lim_{x\to 4} f = \frac{5}{\sqrt{8}} = 1,8$

• $\lim_{x\to 0} f = \frac{5}{\sqrt{4}} = 2,5$

• $\lim_{x\to +\infty} f = \frac{5}{+\infty} = 0 \quad \left(\frac{\text{grado } 0}{\text{grado } 0,5}\right)$

• $\lim_{x\to -\infty} f = \nexists$

Geogebra

Corona Límites 3

① $f(x)=e^{\sqrt{x}}$

- $\lim_{x\to -1} f = e^{\sqrt{-1}} = \nexists$
- $\lim_{x\to 1} f = e^{\sqrt{1}} = e^{1} = e$
- $\lim_{x\to 0} f = e^{\sqrt{0}} \longrightarrow \lim_{x\to 0^-} f = e^{\sqrt{-0}} = \nexists$

  $\longrightarrow \lim_{x\to 0^+} f = e^{\sqrt{+0}} = e^{0} = 1$
- $\lim_{x\to +\infty} f = e^{\sqrt{+\infty}} = e^{+\infty} = +\infty$
- $\lim_{x\to -\infty} f = e^{\sqrt{-\infty}} = \nexists$

② $f(x)=e^{1/x^2}$

- $\lim_{x\to -1} f = e^{1} = e$
- $\lim_{x\to 1} f = e^{1} = e$
- $\lim_{x\to 0} f = e^{1/0} = e^{\pm\infty} \longrightarrow \lim_{x\to 0^+} f = e^{+\infty} = +\infty$

  $\longrightarrow \lim_{x\to 0^-} f = e^{+\infty} = +\infty$
- $\lim_{x\to +\infty} f = e^{\frac{1}{+\infty}} = e^{0} = 1$; • $\lim_{x\to -\infty} f = e^{0} = 1$

③ $e^{\frac{x^2}{x^2-1}}$

- $\lim_{x\to -1} f = e^{\frac{1}{0}} = e^{\pm\infty}$

  $\longrightarrow \lim_{x\to -1^-} f = e^{\frac{1}{+0}} = e^{+\infty} = +\infty$

  $\longrightarrow \lim_{x\to -1^+} f = e^{\frac{1}{-0}} = e^{-\infty} = 0$
- $\lim_{x\to 1} f = e^{\frac{1}{0}} = e^{\pm\infty}$

  $\longrightarrow \lim_{x\to 1^+} f = e^{\frac{1}{+0}} = e^{+\infty} = +\infty$

  $\longrightarrow \lim_{x\to 1^-} f = e^{\frac{1}{-0}} = e^{-\infty} = 0$
- $\lim_{x\to 0} f = e^{\frac{0}{-1}} = e^{0} = 1$
- $\lim_{x\to +\infty} f = e^{\frac{+\infty}{+\infty}} = e^{1} = e$
- $\lim_{x\to -\infty} f = e^{\frac{+\infty}{+\infty}} = e^{1} = e$

Geogebra

④ $f(x) = \cancel{e^x}\ \ln(\sqrt{x})$

Corona Límites 3

- $\lim_{x\to -1} f = \ln(\sqrt{-1}) = \nexists$
- $\lim_{x\to 1} f = \ln(\sqrt{1}) = \ln 1 = 0$
- $\lim_{x\to 0} f = \ln(\sqrt{0}) = \ ?$
  - $\to \lim_{x\to 0^-} f = \ln(\sqrt{-0}) = \nexists$
  - $\to \lim_{x\to 0^+} f = \ln(\sqrt{+0}) = \ln(0) = -\infty$
- $\lim_{x\to +\infty} f = \ln(\sqrt{+\infty}) = \ln(+\infty) = +\infty$
- $\lim_{x\to -\infty} f = \ln(\sqrt{-\infty}) = \nexists$

⑤ $f(x) = \ln\left(\frac{1}{x^2}\right)$

- $\lim_{x\to \pm 1} f = \ln(1) = 0$
- $\lim_{x\to 0^{+-}} f = \ln(+\infty) = +\infty$
- $\lim_{x\to \pm\infty} f = \ln(+0) = -\infty$

⑥ $f(x) = \ln\left(\frac{x^2}{x^2-1}\right)$

Corona Límites 3

- $\lim_{x\to -1} f = \ln\left(\frac{1}{0}\right) = \ln(\pm\infty)$
  - $\to \lim_{x\to -1^-} f = \ln\left(\frac{1}{+0}\right) = \overset{\ln(+\infty)}{\cancel{+\infty}} = +\infty$
  - $\to \lim_{x\to -1^+} f = \ln\left(\frac{1}{-0}\right) = \ln(-\infty) = \nexists$
- $\lim_{x\to 1} f = \ln\left(\frac{1}{0}\right) = \ln(\pm\infty)$
  - $\to \lim_{x\to 1^-} f = \ln\left(\frac{1}{-0}\right) = \ln(-\infty) = \nexists$
  - $\to \lim_{x\to 1^+} f = \ln\left(\frac{1}{+0}\right) = \ln(+\infty) = +\infty$
- $\lim_{x\to \pm\infty} f = \ln\left(\frac{+\infty}{+\infty}\right) = \ln(1) = 0$

Coronalímites 4

① $x^2-x-6=0 \longrightarrow x = \begin{cases} -2 \\ 3 \end{cases}$ Dom f $= \mathbb{R}-\{2,3\}$

Ec. 2º grado

- Asint. Vert $\lim_{x\to -2} f = \left(\frac{-2}{\pm 0}\right) = \pm\infty \longrightarrow \boxed{\text{Asíntota } x=-2}$

$\lim_{x\to 3} f = \left(\frac{3}{\pm 0}\right) = \pm\infty \longrightarrow \boxed{\text{Asíntota } x=3}$

- Asint. Hor. $\lim_{x\to\pm\infty} f = \left|\frac{\pm\infty}{\pm\infty}\right| = 0$ (grados) $\longrightarrow \boxed{\text{Asíntota } y=0}$

- Asint. Obl.: No

② Dom f $= \mathbb{R}-\{-2,3\}$

- A. Ver: $\lim_{x\to -2} f = \left(\frac{8}{\pm 0}\right) = \pm\infty \longrightarrow \boxed{\text{Asíntota } x=-2}$

$\lim_{x\to 3} f = \left|\frac{18}{\pm 0}\right| = \pm\infty \longrightarrow \boxed{\text{Asíntota } x=3}$

- A. Hor: $\lim_{x\to\pm\infty} f = \cancel{\frac{\pm\infty}{\pm\infty}} = \left(\frac{+\infty}{+\infty}\right) = 2$ (grados) $\longrightarrow \boxed{\text{Asíntota } y=2}$

- A. Obl: No

Coronalímites 4

Geogebra

③ Dom f $= \mathbb{R}-\{-2,3\}$

Asint. Vert: $\lim_{x\to -2} f = \frac{-16}{\pm\infty} = \cancel{\pm\infty}\ 0$ $\boxed{\text{A.V. } x=-2}$

$\lim_{x\to 3} f = \frac{54}{\pm\infty} = 0$ $\boxed{\text{A.V. } x=3}$

Asint. Hor: $\lim_{x\to\pm\infty} f = \frac{\pm\infty}{+\infty} = \pm\infty \longrightarrow$ No

Asint. Obl: $\frac{f(x)}{x} = \frac{2x^3}{x^3-x^2-6x}$; $\lim_{x\to\pm\infty} \frac{f(x)}{x} = 2$

$m=2$

$f(x)-mx = \frac{2x^3}{x^2-x-6} - 2x = \frac{2x^3-2x^3+2x^2+12x}{x^2-x-6} =$

$= \frac{2x^2+12x}{x^2-x-6}$; $\lim_{x\to\pm\infty} (f-mx) = 2$

$n=2$

$\boxed{\text{A.O: } y = 2x+2}$

Corona Límites 4

(4) Dominio $2x^3-14x+12=0$ Ruffini

$$\begin{array}{r|rrrr} & 2 & 0 & -14 & 12 \\ 1 & & 2 & 2 & -12 \\ \hline & 2 & 2 & -12 & \boxed{0} \\ 2 & & 4 & 12 & \\ \hline & 2 & 6 & \boxed{0} & \\ -3 & & -6 & & \\ \hline & 2 & \boxed{0} & & \end{array}$$

$\longrightarrow x=1$

$\longrightarrow x=2$

$\longrightarrow x=-3$

$2(x-1)(x-2)(x+3)$

$Dom\, f = \mathbb{R}-\{1,2,-3\}$

• As. Ver: • $\lim_{x\to 1} f = \left(\frac{0}{0}\right) \to$ Simplificar

$f(x) = \frac{\cancel{2(x-1)}}{\cancel{2(x-1)}(x-2)(x+3)} = \frac{1}{(x-2)(x+3)}$

$\lim_{x\to 1} f = \frac{1}{(1-2)(1+3)} = \frac{1}{-4} \to$ $\boxed{\text{Punto Ab. No Asíntota}}$

• $\lim_{x\to 2} f = \left(\frac{2}{\pm 0}\right) = \pm\infty$ $\boxed{\text{Asin Ver } x=2}$

• $\lim_{x\to -3} f = \left(\frac{-8}{\pm 0}\right) = \pm\infty$ $\boxed{\text{Asin Ver } x=-3}$

• Asint. Hor. $\lim_{x\to\pm\infty} f = \left(\frac{\pm\infty}{\pm\infty}\right) = 0$ $\boxed{\text{A.H } y=0}$ Geogebra

• Asint. Obl: No

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(5) $Dom\, f = \mathbb{R}-\{-3,1,2\}$

• As. Ver: • $\lim_{x\to -3} f = \left(\frac{-35}{\pm 0}\right) = \pm\infty \to$ $\boxed{\text{AV } x=-3}$

• $\lim_{x\to 1} f = \left(\frac{-7}{\pm 0}\right) = \pm\infty \to$ $\boxed{\text{A.V. } x=1}$

• $\lim_{x\to 2} f = \left(\frac{0}{0}\right)$ Simplificar

$\to f(x) = \frac{\cancel{(x-2)}(x^2+2x+4)}{2(x-1)\cancel{(x-2)}(x+3)}$

x^3-8

$$\begin{array}{r|rrrr} & 1 & 0 & 0 & -8 \\ 2 & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & \boxed{0} \end{array}$$

$\to x=2$

No se puede seguir

$(x-2)(x^2+2x+4)$

$\lim_{x\to 2} f = \frac{12}{2(2-1)(2+3)} = \frac{12}{10}$ $\to$ $\boxed{\text{Pto Abierto. No AV}}$

Corona límites 4

• $\cancel{\lim_{x\to -3} f =}$

• Asínt. Hor: $\lim_{x\to\pm\infty} f = \left(\frac{\pm\infty}{\pm\infty}\right) = \frac{1}{2}$ $\boxed{A.H.\ y=\frac{1}{2}}$

• Asínt. Obl: No

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⑥ $f(x) = \dfrac{x^4-16}{2x^3-14x+12}$ Dom $f = \mathbb{R} - \{-3, 1, 2\}$

• As. Ve: • $\lim_{x\to -3} f = \left(\frac{65}{\pm 0}\right) = \pm\infty$ $\boxed{A.V.\ x=-3}$

• $\lim_{x\to 1} f = \left(\frac{-15}{\pm 0}\right) = \pm\infty$ $\boxed{A.V.\ x=1}$

• $\lim_{x\to 2} f = \left(\frac{0}{0}\right)$

$x^4-16=0$

$$\begin{array}{r|rrrrr} & 1 & 0 & 0 & 0 & -16 \\ 2 & & 2 & 4 & 8 & 16 \\ \hline & 1 & 2 & 4 & 8 & \boxed{0} \\ -2 & & -2 & 0 & -8 & \\ \hline & 1 & 0 & 4 & \boxed{0} & \end{array}$$

$x=2$

$\to x=-2$

No Hay más

$(x-2)(x+2)(x^2+4)$

$f(x) = \dfrac{\cancel{(x-2)}(x+2)(x^2+4)}{2(x+3)(x-1)\cancel{(x-2)}}$; $\lim_{x\to 2} f = \dfrac{(2+2)(4+4)}{2(5)(1)} = \dfrac{32}{10}$

Geogebra

Punto Abierto $\to$ $\boxed{\text{No}\ \text{As. Ver}\ x=2}$

• As. Hor ; $\lim_{x\to\pm\infty} f = \left(\frac{\pm\infty}{\pm\infty}\right) = \pm\infty \to$ No

• As. Obl ; $\dfrac{f(x)}{x} = \dfrac{x^4-16}{2x^4-14x^2+12x}$

$\lim_{x\to\pm\infty} \dfrac{f(x)}{x} = \dfrac{1}{2}$ $m = \frac{1}{2}$

$\cancel{f(x) - mx = \dfrac{x^4-16}{2x^3-14x^2+12x} - \dfrac{x}{2} = \dfrac{2x^4-32-2x}{4x^4-28x^2+24x}}$

$f(x) - mx = \dfrac{x^4-16}{2x^3-14x+12} - \dfrac{x}{2} = \dfrac{\cancel{2x^4}-32-\cancel{2x^4}+14x^2-12x}{4x^3-28x+24}$

$\lim_{x\to\pm\infty} (f(x)-mx) = \left(\frac{\pm\infty}{\pm\infty}\right) = 0$ (grados)

$n=0$

A. O. : $\boxed{y = \frac{1}{2}x}$

Geogebra

Corona Límites 5

$f(x) = \dfrac{\sqrt{x}-3}{x-3}$ $\quad$ Dom $f = [0,+\infty) - \{3\}$

• A. Vert : • $\lim\limits_{x\to 0^+} f = \dfrac{-3}{-3} = 1$ $\quad$ Punto

• $\lim\limits_{x\to 3} f = \left(\dfrac{\sqrt{3}-3}{0}\right) = \pm\infty$ $\quad$ A.V.

• A. Hor : • $\lim\limits_{x\to +\infty} f = \left(\dfrac{+\infty}{+\infty}\right) = 0$ (grados)

$f(x) = \dfrac{\sqrt{x}-3}{x}$ $\quad$ Dom $f = (0,+\infty)$

• A. Vert : • $\lim\limits_{x\to 0^+} f = \left(\dfrac{-3}{+0}\right) = -\infty$ $\quad$ A.V.

• A. Hor : • $\lim\limits_{x\to +\infty} f = \left(\dfrac{+\infty}{+\infty}\right) = 0$ (grados)

$f(x) = \dfrac{x}{\sqrt{x}-3}$ $\quad$ Dom $f = [0,+\infty) - \{9\}$

• A. Vert $\quad$ • $\lim\limits_{x\to 0} f = \dfrac{0}{-3} = 0$ $\quad$ Punto

• $\lim\limits_{x\to 9} f = \left(\dfrac{9}{0}\right) = \pm\infty$ $\quad$ A.V.

• A. Hor $\quad$ • $\lim\limits_{x\to +\infty} f = \left(\dfrac{+\infty}{+\infty}\right) = +\infty$ (grados)

• A. Oblic $\quad \longrightarrow$

Corona Límites 5

$\dfrac{f(x)}{x} = \dfrac{\frac{x-3}{\sqrt{x}-3}}{x} = \dfrac{x-3}{x(\sqrt{x}-3)} = \dfrac{x-3}{x\sqrt{x}-3x}$

$\lim\limits_{x\to +\infty} \dfrac{f(x)}{x} = \left(\dfrac{+\infty}{+\infty}\right) = 0 \left(\dfrac{\text{grado } 1}{\text{grado } 1,5}\right)$

$m = 0 \to$ Imposible, debe ser oblicua $\to$

$\to$ No hay asínt. oblic

$f(x) = e^{1-\frac{1}{x}}$ $\quad$ Dom $f = \mathbb{R} - \{0\}$

• A. Vert $\quad$ • $\lim\limits_{x\to 0} f = e^{1-\frac{1}{0}} = e^{1-(\pm\infty)}$

• $\lim\limits_{x\to 0^-} f = e^{1-\left(\frac{1}{-0}\right)} = e^{(+\infty)} = +\infty$

• $\lim\limits_{x\to 0^+} f = e^{1-\left(\frac{1}{+0}\right)} = e^{(-\infty)} = 0$

• A. Hor $\quad$ • $\lim\limits_{x\to +\infty} f = e^{1-\left(\frac{1}{+\infty}\right)} = e^{1-0} = e$

A.H. $\quad$ • $\lim\limits_{x\to -\infty} f = e^{1-\left(\frac{1}{-\infty}\right)} = \cancel{e^{1+\infty} = e^{+\infty} = +\infty}$

$= e^{1+0} = e$

~~No~~ A.H.

Corona Límites 5

Geogebra

$f(x) = \ln\left(1 - \frac{1}{x}\right)$

Dominio: $1 - \frac{1}{x} > 0$ $\qquad 1 - \frac{1}{x} = 0$

$x = 1$

$+ \quad 0 \quad - \quad 1 \quad +$

$\text{Dom } f = (-\infty, 0) \cup (1, +\infty)$

• A. Vert • $\lim\limits_{x \to 0^-} f = \ln\left(1 - \frac{1}{-0}\right) = \ln(+\infty) = +\infty$

A.V. $\longleftarrow$

• $\lim\limits_{x \to 1^+} f = \ln(+0) = -\infty \longrightarrow$ A.V.

A. Hor: $\lim\limits_{x \to +\infty} f = \ln(1 - 0) = 0$

$\lim\limits_{x \to -\infty} f = \ln(1 + 0) = 0$

$>$ A.H.