Ficha 9.1.

(1) $\int (3x^3 - 5x^2 + 3x + 4)\,dx =$

$= 3\frac{x^3}{3} - 5\frac{x^3}{3} + 3\frac{x^2}{2} + 4x =$

$= x^3 - \frac{5x^3}{3} + \frac{3x^2}{2} + 4x + k$

(2) $\int (\operatorname{sen} x + 7\cos x - 1)\,dx =$

$= -\cos x + 7\operatorname{sen} x - x + k$

(3) $\int (\sqrt{x} - 2)\,dx = \int (x^{1/2} - 2)\,dx =$

$= \frac{x^{3/2}}{3/2} - 2x = \frac{2}{3}\sqrt{x^3} - 2x + k$

(4) $\int \frac{2}{\sqrt{x}}\,dx = 2\int x^{-1/2}\,dx = 2\frac{x^{1/2}}{1/2} = 4\sqrt{x} + k$

(5) $\int \frac{x^3 - 2x^2 + 4x}{x}\,dx = \int (x^2 - 2x + 4)\,dx =$

$= \frac{x^3}{3} - \frac{2x^2}{2} + 4x = \frac{x^3}{3} - x^2 + 4x + k$

Ficha 9.1.

(6) $\int \frac{(2x-1)^2}{2x}\,dx = \int \frac{4x^2 - 4x + 1}{2x}\,dx =$

$= \int \left(2x - 2 + \frac{1}{2x}\right)dx = 2\int x\,dx - 2\int dx + \frac{1}{2}\int \frac{1}{x}\,dx$

$= 2\frac{x^2}{2} - 2x + \frac{1}{2}\ln|x| = x^2 - 2x + \frac{1}{2}\ln|x| + k$

(7) $\int (2\sqrt{x} - \sqrt[3]{x} - x^4)\,dx = \int (2x^{1/2} - x^{1/3} - x^4)\,dx =$

$= 2\frac{x^{3/2}}{3/2} - \frac{x^{4/3}}{4/3} - \frac{x^5}{5} =$

$= \frac{4}{3}\sqrt[2]{x^3} - \frac{3}{4}\sqrt[3]{x^4} - \frac{x^5}{5} + k$

(8) $\int \left(\frac{3}{x} - \frac{x}{3}\right)dx = 3\int \frac{1}{x}\,dx - \frac{1}{3}\int x\,dx =$

$= 3\ln|x| - \frac{1}{3}\frac{x^2}{2} = 3\ln|x| - \frac{x^2}{6} + k$

(9) $\int \frac{2e^x + e^{2x}}{e^x}\,dx = \int (2 + e^x)\,dx =$

$= 2x + e^x + k$

Ficha 9.1

(10) $\int \frac{2}{1+x^2}\,dx = 2\operatorname{arctg} x + k$

(11) $\int (4x+2)(x-1)\,dx = \int (4x^2 - 2x - 2)\,dx =$

$= 4\frac{x^3}{3} - 2\frac{x^2}{2} - 2x = \frac{4x^3}{3} - x^2 - 2x + k$

(12) $\int \frac{\cancel{x+2}}{2\cancel{\sqrt{x+2}}}\,dx = \int \frac{\sqrt{x+2}}{2}\,dx =$

$= \frac{1}{2}\int (x+2)^{1/2}\,dx = \frac{1}{2}\frac{(x+2)^{3/2}}{3/2} =$

$= \frac{1}{3}\sqrt[2]{x+2}^{\,3} + k$

(13) $\int 5^x\,dx = \frac{5^x}{\ln 5} + k$

Ficha 9.2.

(1) $\int (x+\sqrt{x})\,dx = \int (x^1 + x^{1/2})\,dx =$

$= \frac{x^2}{2} + \frac{x^{3/2}}{3/2} = \frac{x^2}{2} + \frac{2\sqrt{x^3}}{3} + k$

(2) $\int \cos(2x)\,dx = \frac{1}{2}\int 2\cos(2x)\,dx =$

$= \frac{1}{2}\,\text{sen}(2x) + k$

(3) $\int \frac{2}{3x+2}\,dx = 2\,\frac{1}{3}\int \frac{3}{3x+2}\,dx = \frac{2}{3}\ln|3x+2| + k$

(4) $\int e^{2x}\,dx = \frac{1}{2}\int 2e^{2x}\,dx = \frac{1}{2}e^{2x} + k$

(5) $\int \frac{1}{1-x}\,dx = (-1)\int \frac{(-1)}{1-x}\,dx = (-1)\ln|1-x| + k$

(6) $\int e^{5x+3}\,dx = \frac{1}{5}\int 5e^{5x+3}\,dx = \frac{e^{5x+3}}{5} + k$

(7) $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4}\right)dx = 3\int x^{-1/2}\,dx - \frac{1}{4}\int x^{3/2}\,dx =$

$= 3\,\frac{x^{1/2}}{1/2} - \frac{1}{4}\,\frac{x^{5/2}}{5/2} = 6\sqrt{x} - \frac{\sqrt{x^5}}{10} + k$

(8) $\int \left(\frac{1}{\sqrt[5]{x}} + \frac{5}{e^{3x}}\right)dx = \int x^{-1/5}\,dx + 5\int e^{-3x}\,dx -$

$= \int x^{-1/5}\,dx - \frac{5}{3}\int (-3)e^{-3x}\,dx =$

$= \frac{x^{4/5}}{4/5} - \frac{5}{3}e^{-3x} = \frac{5}{4}\sqrt[5]{x^4} - \frac{5}{3}e^{-3x} + k$

(9) $\int x\,\text{sen}(x^2)\,dx = \frac{1}{2}\int 2x\,\text{sen}(x^2)\,dx =$

$= -\frac{1}{2}\cos(x^2) + k$

(10) $\int \frac{x^3}{1+x^4}\,dx = \frac{1}{4}\int \frac{4x^3}{1+x^4}\,dx = \frac{1}{4}\ln|1+x^4|$

(11) $\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int \frac{2x}{1+(x^2)^2}\,dx = \frac{1}{2}\,\text{arc tg}\,x^2$

(12) $\int \sqrt{x+1}\,dx = \int (x+1)^{1/2}\,dx = \frac{(x+1)^{3/2}}{3/2} =$

$= \frac{2\sqrt{(x+1)^3}}{3} + k$

(13) $3\,\frac{1}{2}\int 2x(x^2+3)^{-1/3}\,dx = \frac{3}{2}\,\frac{(x^2+3)^{2/3}}{2/3} =$

$= \frac{9}{4}\sqrt[3]{(x^2+3)^2} + k$

Ficha 9.3

(1) $\int 3\sqrt{5x}\,dx = 3\sqrt{5}\int x^{1/2}\,dx = 3\sqrt{5}\,\frac{x^{3/2}}{3/2} =$

$= \frac{3\sqrt{5}\,2}{\cancel{3}}\sqrt{x^3} = 2\sqrt{5}\sqrt{x^3} + k$

(2) $\int \frac{8}{x} + \sqrt[3]{2x}\,dx = 8\int \frac{1}{x}\,dx + \sqrt[3]{2}\int x^{1/3}\,dx =$

$= 8\ln|x| + \sqrt[3]{2}\,\frac{x^{4/3}}{4/3} = 8\ln|x| + \frac{3\sqrt[3]{2}}{4}\sqrt[3]{x^4} + k$

(3) $\int \cos(5x+1)\,dx = \frac{1}{5}\int 5\cos(5x+1)\,dx =$

$= \frac{1}{5}\,\text{sen}(5x+1) + k$

(4) $\int \frac{1}{\sqrt{x+2}^{\,3}}\,dx = \int (x+2)^{-3/2}\,dx =$

$= \frac{(x+2)^{-1/2}}{-1/2} = \cancel{-\frac{2}{3}\sqrt[3]{\ }} = -2\cancel{\sqrt{x+2}} + k$

$= -2\,\frac{1}{\sqrt{x+2}} + k$

(5) $\int \frac{4}{x-2}\,dx = 4\ln|x-2| + k$

(6) $\int \left(\frac{1}{x^2+1} - \frac{2}{x^2} + \frac{3}{x}\right)dx = \text{arc tg}\,x + \frac{2}{x} + 3\ln|x|$

$(1+x^2) \quad x^{-2}$

(7) $\int \frac{6x}{x^2+1}\,dx = 6\,\frac{1}{2}\int \frac{2x}{x^2+1}\,dx = 3\ln|x^2+1| + k$

(8) $\int \frac{x}{\sqrt{1+2x^2}}\,dx = \frac{1}{4}\int \frac{4x}{\sqrt{1+2x^2}}\,dx = \frac{1}{4}\int 4x(1+2x^2)^{-1/2}\,dx$

$= \frac{1}{4}\,\frac{(1+2x^2)^{1/2}}{1/2} = \frac{1}{2}\sqrt{1+2x^2} + k$

(9) $\int \frac{x+\sqrt{x}}{x^2}\,dx = \int (x^{-1} + x^{-3/2})\,dx =$

$= \ln|x| + \frac{x^{-1/2}}{-1/2} = \ln|x| - 2\,\frac{1}{\sqrt{x}} + k$

(10) $\int \cos x\,\text{sen}^3 x\,dx = \frac{\text{sen}^4 x}{4} + k$

u^3

(11) $\int \frac{2x}{x^2-1}\,dx = \ln|x^2-1| + k$

(12) $\int 2x\,\text{sen}\,x^2\,dx = -\cos x^2 + k$

(13) $\int \text{sen}\,x\,e^{\cos x} = (-1)\int (-1)\,\text{sen}\,x\,e^{\cos x}\,dx$

$= -e^{\cos x} + k$

Ficha 9.4.

(1) $\int \frac{1}{4+x^2}dx = \int \frac{\frac{1}{4}}{\frac{4}{4}+\frac{x^2}{4}}dx = \frac{1}{4}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}dx$

$= \frac{1}{\underset{2}{\cancel{4}}}\cdot\cancel{2}\cdot\int \frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2}dx = \frac{1}{2}\text{ arc tg}\left(\frac{x}{2}\right)+k$

(2) $\int \frac{1}{3+x^2}dx = \int \frac{\frac{1}{3}}{1+\left(\frac{x}{\sqrt{3}}\right)^2} = \frac{1}{3}\sqrt{3}\int \frac{\frac{1}{\sqrt{3}}}{1+\left(\frac{x}{\sqrt{3}}\right)^2}dx$

$= \frac{\sqrt{3}}{3}\text{ arc tg}\frac{x}{\sqrt{3}}+k$

(3) $\int \frac{1}{\sqrt{9-x^2}}dx = \int \frac{\frac{1}{3}}{\sqrt{\frac{9}{9}-\frac{x^2}{9}}}dx = \frac{1}{3}\int \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}}dx$

$= \frac{1}{3}\cdot 3\cdot\int \frac{\frac{1}{3}}{\sqrt{1-\left(\frac{x}{3}\right)^2}}dx = 1\cdot\text{arc sen}\frac{x}{3}+k$

(4) $\int \frac{1}{\sqrt{1-9x^2}}dx = \int \frac{1}{\sqrt{1-(3x)^2}}dx = \frac{1}{3}\int \frac{3}{\sqrt{1-(3x)^2}}dx$

$= \frac{1}{3}\text{ arc sen}(3x)+k$

(5) $\int \frac{5x}{1+x^4}dx = 5\int \frac{x}{1+\underbrace{(x^2)^2}_{u^2}}dx =$

$= 5\cdot\frac{1}{2}\cdot\int \frac{2x}{1+(x^2)^2}dx = \frac{5}{2}\text{ arc tg }x^2+k$

(6) $\int \frac{x}{\sqrt{1-x^4}}dx = \int \frac{x}{\sqrt{1-(x^2)^2}}dx = \frac{1}{2}\int \frac{2x}{\sqrt{1-(x^2)^2}}dx$

$= \frac{1}{2}\text{ arc sen }x^2+k$

(7) $\int \frac{dx}{1+2x^2} = \int \cancel{\frac{1}{2}}\,dx = \int \frac{1}{1+(\sqrt{2}x)^2}dx =$

$= \frac{1}{\sqrt{2}}\int \frac{\sqrt{2}}{1+(\sqrt{2}x)^2}dx = \frac{1}{\sqrt{2}}\text{ arc tg}(\sqrt{2}x)+k$

(8) $\int \frac{x}{x^2+1}dx = \frac{1}{2}\int \frac{2x}{x^2+1}dx = \frac{1}{2}\ln|x^2+1| + k$

(9) $\int \frac{dx}{5-2x} = \left(\frac{1}{-2}\right)\int \frac{-2}{5-2x}dx = -\frac{1}{2}\ln|5-2x| + k$

Ficha 9.4

(10) $\int e^{2x}dx = \frac{1}{2}\int 2e^{2x}dx = \frac{1}{2}e^{2x}+k$

(11) $\int e^{\frac{x}{2}}dx = 2\int \frac{1}{2}e^{\frac{x}{2}}dx = 2e^{\frac{x}{2}}+k$

(12) $\int e^{-3x}dx = -\frac{1}{3}\int -3e^{-3x}dx = -\frac{1}{3}e^{-3x}+k$

(13) $\int \frac{e^x}{3+4e^x}dx = \frac{1}{4}\int \frac{4e^x}{3+4e^x}dx =$

$= \frac{1}{4}\ln|3+4e^x|+k$

138 respuestas ✓

Ficha 9.5

(1) $\int x(2x+5)^{10}dx \quad \left[\begin{matrix} u=2x+5\,;\ x=\frac{u-5}{2} \\ du=2dx\,;\ dx=\frac{du}{2}\end{matrix}\right]$

$\int \frac{u-5}{2}\cdot u^{10}\cdot\frac{du}{2} = \frac{1}{2}\cdot\frac{1}{2}\int(u^{11}-5u^{10})\,dx =$

$= \frac{1}{4}\left(\frac{u^{12}}{12}-5\frac{u^{11}}{11}\right) = \frac{1}{4}\left(\frac{(2x+5)^{12}}{12}-\frac{5(2x+5)^{11}}{11}\right)+k$

(2) $\int \frac{\ln x}{x}dx \quad \left[\begin{matrix} u=\ln x \\ du=\frac{1}{x}dx\end{matrix}\right]$

$\int u\cdot du = \frac{u^2}{2} = \frac{(\ln x)^2}{2}+k$

(3) $\int \frac{\cos x}{\operatorname{sen}^2 x}dx = \left[\begin{matrix} u=\operatorname{sen} x \\ du=\cos x\,dx\end{matrix}\right]$

$= \int u^{-2}du = \frac{u^{-1}}{-1} = -\frac{1}{u} = -\frac{1}{\operatorname{sen}x}+k$

(4) $\int \frac{dx}{x+\sqrt{x}} \quad \left[\begin{matrix} u=\sqrt{x} \quad u^2=x \\ 2u\,du=dx\end{matrix}\right] =$

$= \int\frac{2u\,du}{u^2+u} = 2\int\frac{u}{u^2+u}du = 2\int\frac{1}{u+1}du$

$= 2\ln|u+1| = 2\ln|\sqrt{x}+1|+k$

(5) $\cancel{\int\frac{e^{2x}}{2+e^x}dx \quad \left[\begin{matrix} e^x=u \\ e^x dx=du \\ dx=\frac{du}{u}\end{matrix}\right] = \int\frac{u^2}{2+u}\frac{du}{u} = \int\frac{u}{2+u}du}$

(5) $\int\frac{e^{2x}}{2+e^x}dx \quad \left[\begin{matrix} 2+e^x=u\,;\ e^x=u-2 \\ e^x dx=du\,;\ dx=\frac{du}{u-2}\end{matrix}\right]$

$\int\frac{(u-2)^2}{u}\frac{du}{u-2} = \int\frac{u-2}{u}du =$

$= \int\left(1-\frac{2}{u}\right)du = u-2\ln|u| =$

$= 2+e^x-2\ln|2+e^x|+k$

(6) $\int x\sqrt[3]{x-4}\,dx = \left[\begin{matrix} x-4=u\,;\ x=u+4 \\ dx=du\end{matrix}\right]$

$= \int(u+4)u^{1/3}du = \int(u^{\frac{4}{3}}+4u^{1/3})du =$

$= \frac{u^{7/3}}{7/3}+4\frac{u^{4/3}}{4/3} = \frac{3\sqrt[3]{(x-4)^7}}{7}+3\sqrt[3]{(x-4)^4}+k$

Ficha 9.5

(7) $\cancel{\int\frac{dx}{(x+1)\sqrt{x}}}$

(8) $\int\frac{\sqrt{x}}{\sqrt{1-x^3}}dx \quad \left[\begin{matrix} \sqrt{x}=u\,;\ x=u^2 \\ dx=2u\,du\end{matrix}\right]$

$\int\frac{u}{\sqrt{1-u^6}}2u\,du = 2\int\frac{u^2}{\sqrt{1-(u^3)^2}}du =$ (arc sen)

$= 2\frac{1}{3}\int\frac{3u^2}{\sqrt{1-(u^3)^2}}du = \frac{2}{3}\operatorname{arc\,sen} u^3 =$

$= \frac{2}{3}\operatorname{arc\,sen}\sqrt{x}^3+k$

(9) $\int\frac{dx}{x(1+\ln x)^3} \quad \left[\begin{matrix} u=\ln x \\ du=\frac{dx}{x}\end{matrix}\right] =$

$= \int\frac{1}{(1+u)^3}du = \int(1+u)^{-3}du =$

$= \cancel{\frac{(1+u)^{-2}}{-2}}\ \frac{(1+u)^{-2}}{-2} = -\frac{1}{2(1+\ln x)^2}+k$

(10) $\int\frac{dx}{x\sqrt{1-(\ln x)^2}} = \left[\begin{matrix} u=\ln x \\ du=\frac{dx}{x}\end{matrix}\right]$

$= \int\frac{1}{\sqrt{1-u^2}}du = \operatorname{arc\,sen} u =$

$= \operatorname{arc\,sen}(\ln x)+k$

Ficha 9.6

(1a) $\int x\sqrt{x+1}\,dx \quad \left[\begin{matrix} u=x+1 \,;\, du=dx \\ u-1=x \end{matrix}\right]$

$$\int (u-1)u^{1/2}\,du = \int (u^{3/2}-u^{1/2})\,du =$$
$$= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} = \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + k$$

(1b) $\int \frac{dx}{x-\sqrt[4]{x}} \,;\, \left[\begin{matrix} u=\sqrt[4]{x} \\ u^4=x \,;\, 4u^3du=dx \end{matrix}\right]$

$$\int \frac{4u^3\,du}{u^4-u} = \int \frac{4u^3}{u(u^2-1)}\,du = 4\int \frac{u^2}{u^3-1}\,du =$$
$$= 4\,\frac{1}{3}\int \frac{3u^2}{u^3-1}\,du = \frac{4}{3}\ln|u^3-1| =$$
$$= \frac{4}{3}\ln|\sqrt[4]{x}^3-1| + k$$

(1c) $\int \frac{x}{\sqrt{x+1}}\,dx \quad \left[\begin{matrix} u=x+1 \,;\, x=u-1 \\ dx=du \end{matrix}\right]$

$$\int \frac{u-1}{u^{1/2}}\,du = \int (u^{1/2}-u^{-1/2})\,du =$$
$$= \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} = \frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + k$$

(1d) $\int \frac{1}{x\sqrt{x+1}}\,dx \quad \cancel{\left[\begin{matrix} u=x+1 \\ du=dx \end{matrix}\right] = \int \frac{1}{(u-1)}\,du}$

$\left[\begin{matrix} u=\sqrt{x+1} \,;\, x=u^2-1 \\ u^2=x+1 \,;\, 2u\,du=dx \end{matrix}\right]$

$$\int \frac{2u\,du}{(u^2-1)u} = 2\int \frac{1}{u^2-1}\,du$$

(1e) $\int \frac{1}{x+\sqrt{x}}\,dx \quad \left[\begin{matrix} u=\sqrt{x} \,;\, u^2=x \\ u^2=x \,;\, 2u\,du=dx \end{matrix}\right]$

$$\int \frac{2u\,du}{u^2+u} = 2\int \frac{1}{u+1}\,du = 2\ln|u+1| =$$
$$= 2\ln|\sqrt{x}+1| + k$$

(1f) $\int \frac{\sqrt{x}}{1+x}\,dx \quad \left[\begin{matrix} u=\sqrt{x} \quad u^2=x \\ dx=2u\,du \end{matrix}\right] =$

$$= \int \frac{u}{1+u^2}\,2u\,du = 2\int \frac{u^2}{1+u^2}\,du$$

Ficha 9.6

(2a) $\int \frac{1}{1+e^x}\,dx \quad \left[\begin{matrix} e^x=u \\ e^x dx=du \,;\, u\,dx=du \\ dx=\frac{du}{u} \end{matrix}\right]$

$$\int \frac{1}{1+u}\,\frac{du}{u} = \int \frac{1}{u+u^2}\,du.$$

(2b) $\int \frac{x+3}{\sqrt{9-x^2}}\,dx = \int \frac{x}{\sqrt{9-x^2}}\,dx + \int \frac{3}{\sqrt{9-x^2}}\,dx =$

$$= -\frac{1}{2}\int (-2x)(9-x^2)^{-1/2}\,dx + \int \frac{\frac{3}{3}}{\sqrt{\frac{9}{9}-\left(\frac{x}{3}\right)^2}}\,dx =$$
$$= \frac{1}{\cancel{2}}\frac{(9-x^2)^{1/2}}{\cancel{1/2}} + \operatorname{arcsen}\left(\frac{x}{3}\right) + k$$

(2c) $\int \frac{dx}{e^{2x}-3e^x} \quad \left[\begin{matrix} u=e^x \\ du=e^x dx \,;\, \frac{du}{u}=dx \end{matrix}\right] =$

$$= \int \frac{1}{u^2-3u}\,\frac{du}{u} = \int \frac{1}{u^3-3u^2}\,du$$

(2d) $\int \frac{\operatorname{sen}(\operatorname{tg} x)}{\cos^2 x}\,dx \quad \left[\begin{matrix} \operatorname{tg} x=u \\ \frac{1}{\cos^2 x}\,dx=du \end{matrix}\right]$

$$= \int \operatorname{sen} u\,du = -\cos u = -\cos(\operatorname{tg} x) + k$$

(2e) $\int \frac{e^{3x}-e^x}{e^{2x}+1}\,dx \quad \left[\begin{matrix} e^x=u \\ dx=\frac{du}{u} \end{matrix}\right] = \int \frac{u^3-u}{u^2+1}\,\frac{du}{u} =$

$$= \int \frac{u^3-u}{u^3+u}\,du = \int \frac{u^2-1}{u^2+1}\,du$$

(2f) $\cancel{\int \frac{1}{1+\sqrt{x}}\,dx \quad \left[\begin{matrix} \sqrt{x}=u \\ x=u^2 \quad dx=2u\,du \end{matrix}\right]}$

$$\cancel{\int \frac{1}{1+u}\,2u\,du = 2\int \frac{u}{1+u}\,du}$$

$$\int \frac{1}{1+\sqrt{x}}\,dx = \left[\begin{matrix} 1+\sqrt{x}=u \\ x=(u-1)^2 \,;\, dx=2(u-1)du \end{matrix}\right]$$

$$\int \frac{2(u-1)}{u}\,du = 2\int \left(1-\frac{1}{u}\right)du = 2(u-\ln|u|)$$
$$= 2\left((1+\sqrt{x}) - \ln|1+\sqrt{x}|\right) + k$$