Corona gráficas 1

① $f(x) = -\frac{x^3}{3} + x$ $\quad Dom\, f = \mathbb{R}$

- Corte eje Y: $x=0 \quad y=0$
- Cortes eje X: $\frac{-x^3}{3} + x = 0 \quad -x^3 + 3x = 0$

$x(-x^2+3)=0$

$x=0$

$x=\pm\sqrt{3}$

- Asíntotas: Polinomio, no tiene

$\lim_{x\to+\infty} f = -\infty \qquad \lim_{x\to-\infty} f = +\infty$

- Monotonía

$f'(x) = -\frac{3x^2}{3} + 1 = -x^2 + 1$

$f'=0 \to -x^2+1=0 \to x=\pm 1$

f'	−	+	−
f	↘	↗	↘
	-1	1	

Mínimo $x=-1 \quad y=\frac{-2}{3}$

Máximo $x=1 \quad y=\frac{2}{3}$

- Curvatura

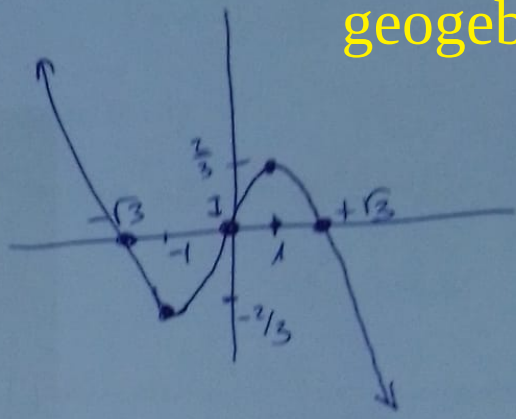
$f''(x) = -6x \to f''=0 \to x=0$

f''	+	0	−
f	∪		∩

P. Infl $x=0 \quad y=0$

Tabla

x	y	
0	0	P. I.
$-\sqrt{3}$	0	
$\sqrt{3}$	0	
$+\infty$	$-\infty$	
$-\infty$	$+\infty$	
-1	$-\frac{2}{3}$	m
1	$\frac{2}{3}$	M



Corona gráficas 1

(2) $f2(x) = \frac{x^3}{12} + \frac{x}{3}$ $\quad Dom f = \mathbb{R}$

- Corte eje Y: $x=0 \quad y=0$
- Corte eje X: $\frac{x^3}{12} + \frac{4x}{12} = 0 \rightarrow x(x^2+4)=0$
  - $x=0$
  - $x=\nexists$
- Asíntotas: Polinómica → No
  - $\lim_{x\to+\infty} f = +\infty$
  - $\lim_{x\to-\infty} f = -\infty$
- Monotonía: $f' = \frac{3x^2}{12} + \frac{1}{3}$

$f'=0 \rightarrow 3x^2+4=0 \rightarrow x=\nexists$

Siempre creciente

f'	$+$
f	$-\infty \nearrow +\infty$

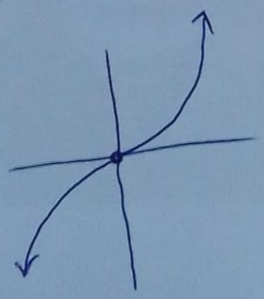
- Curvatura $f'' = \frac{6x}{12} + 0$

$f''=0 \rightarrow \frac{6x}{12}=0 \rightarrow x=0$

f''	$-$	0	$+$
f	∩		∪

P. Inflex $x=0$ $y=0$

x	y
0	0 P.In.
$+\infty$	$+\infty$
$-\infty$	$-\infty$



(3) $f3(x) = x^3 + x^2$ $\quad Dom f = \mathbb{R}$

geogebra

- Cortes eje Y: $x=0 \quad y=0$
- X: $x^3+x^2=0 \rightarrow x^2(x+1)=0$
  - $x=0$
  - $x=-1$
- Asíntotas: No (polinomio) $\lim_{x\to+\infty} f = +\infty$ $\lim_{x\to-\infty} f = -\infty$
- Monotonía: $f'(x) = 3x^2+2x$

$f'=0 \rightarrow 3x^2+2x=0 \rightarrow x(3x+2)=0$

- $x=0$
- $x=-\frac{2}{3}$

f'	$+$	$-\frac{2}{3}$	$-$	0	$+$
f	↗		↘		↗

Máximo en $x=-\frac{2}{3}$ $\quad y=\frac{3}{20}$

Mínimo en $x=0$ $\quad y=0$

- Curvatura $f''(x) = 6x+2$

$f''=0 \rightarrow 6x+2=0 \quad x=-\frac{1}{3}$

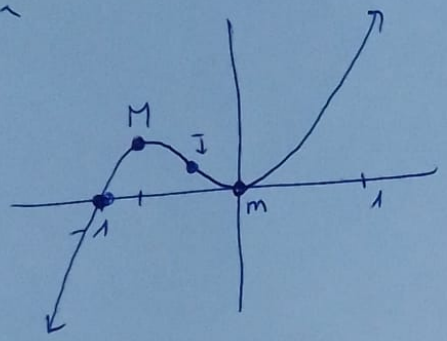
f''	$-$	$-\frac{1}{3}$	$+$
f	∩		∪

P. Inflex $x=-\frac{1}{3}$ $\quad y=\frac{7}{100}$

Corona Gráficas 1

③ Continuación

x	y	
0	0	m
-1	0	
$+\infty$	$+\infty$	
$-\infty$	$-\infty$	
$-\frac{2}{3}$	$\frac{3}{20}$	M
$-\frac{1}{3}$	$\frac{7}{100}$	I

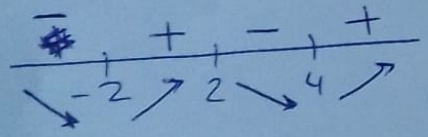


④ $f(x) = 3x^4 - 16x^3 - 24x^2 + 192x - 10$ Dom $f = \mathbb{R}$

- Corte eje Y: $x = 0$ $y = -10$
- Cortes eje X: No lo hacemos esta vez
- Asíntotas: NO $\lim_{x \to +\infty} f = +\infty$ $\lim_{x \to -\infty} f = +\infty$
- Monotonía: $f' = 12x^3 - 48x^2 - 48x + 192$

$f' = 0 \rightarrow x^3 - 4x^2 - 4x + 16 = 0 \rightarrow$ Ruffini

$x = -2$ $x = 2$ $x = 4$



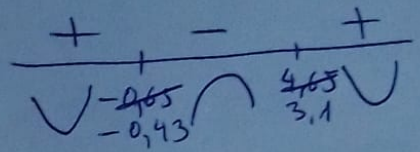
Mínimo $x = -2$ $y = -314$

$x = 4$ $y = 118$

Máximo $x = 2$ $y = 198$

- Curvatura $f'' = 36x^2 - 96x - 48$ geogebra

$f'' = 0$ $x = \frac{4 \pm 2\sqrt{7}}{3} \simeq <$ ~~4,65~~ a_1 3,1 / ~~-0,15~~ -0,43 a_2 -0,43

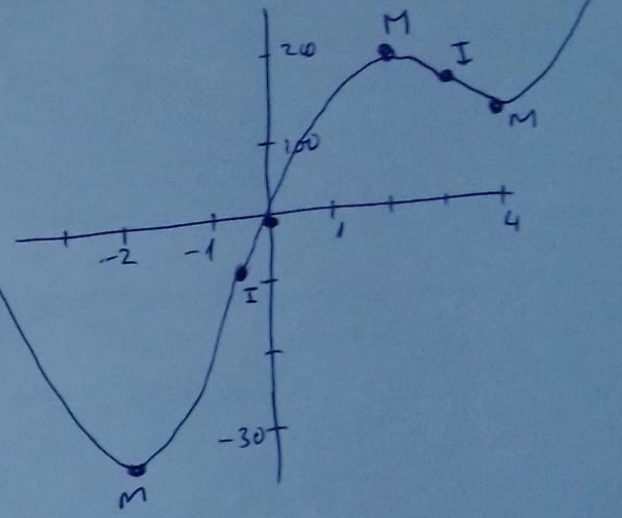


P. Inflex

$x = a_1$ $y = 155,13$

$x = a_2$ $y = -95,72$

x	y	
0	-10	
-2	-314	m
4	118	m
2	198	M
3,1	155	I
-0,4	-96	I
$+\infty$	$+\infty$	
$-\infty$	$+\infty$	



Corona Gráficas 2

1 $f(x) = -6 + 12x - x^3$ Dom $f = \mathbb{R}$

- Corte en Y: $x=0$ $y=-6$
- Asíntotas: No $\lim_{x\to+\infty} f = -\infty$; $\lim_{x\to-\infty} f = +\infty$
- Monotonía: $f' = 12 - 3x^2$

$12 - 3x^2 = 0 \to x = \pm 2$

f'	−	+	−
f	↘	↗	↘
		−2	+2

Mínimo $x=-2$, $y=-22$
Máximo $x=2$, $y=10$

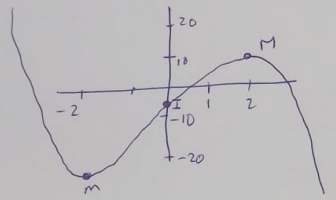
- Curvatura: $f'' = -6x$

$-6x = 0 \to x = 0$

f''	+	−
f	∪	∩
	0	

Pto inflex $x=0$ $y=-6$

x	y	
0	−6	I
$-\infty$	$+\infty$	
$+\infty$	$-\infty$	
−2	−22	m
2	10	M



2 $f(3)(x) = 3x^5 - 50x^3 + 135x$ Dom $f = \mathbb{R}$

- Cortes ejes $x=0$ $y=0$

$y=0$ $x(3x^4 - 50x^2 + 135) = 0$

$x = 0$

$3x^4 - 50x^2 + 135 = 0 \xrightarrow{\text{bicuadrada}} x = \pm 3, \pm 1{,}8$

- Asíntotas: No $\lim_{x\to+\infty} f = +\infty$ $\lim_{x\to-\infty} f = -\infty$
- Monotonía: $f'(x) = 15x^4 - 150x^2 + 135$

$15x^4 - 150x^2 + 135 = 0 \xrightarrow{\text{bicuadrada}} x = \pm 3, \pm 1$

f'	+	−	+	−	+
f	↗	↘	↗	↘	↗
	−3	−1	1	3	

Máx: $x=-3$, $y=216$; $x=1$, $y=88$
Mín: $x=-1$, $y=-88$; $x=3$, $y=-216$

- Curvatura $f''(x) = 60x^3 - 300x$

$60x^3 - 300x = 0$

$x(60x^2 - 300) = 0$

$x = 0$

$x = \pm\sqrt{5} = \pm 2{,}2$

	−	+	−	+
	∩	∪	∩	∪
	−2,2	0	2,2	

Pto inf: $x=0$ $y=0$

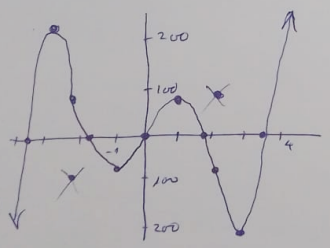
$x=\sqrt{5}$, $y=-89$

$x=-\sqrt{5}$ $y=+89$

Corona Gráficas 2

2

x	y	
0	0	I
$\pm 3{,}6$	0	
$\pm 1{,}8$	0	
$+\infty$	$+\infty$	
$-\infty$	$-\infty$	
−3	216	M
−1	−88	m
1	88	M
+3	−216	m
+2,2	−89	I
−2,8	89	I



3 $f(4)(x) = x^4 - x^3 - x^2 + x$ Dom $f = \mathbb{R}$

- Cortes ejes: $x=0$ $y=0$

$y=0$ $x(x^3 - x^2 - x + 1) = 0$

$x = 0$

$x^3 - x^2 - x + 1 = 0 \xrightarrow{\text{Ruffini}} x = -1, 1$

- Asíntotas: No

$\lim_{x\to\pm\infty} f = +\infty$

3

- Monotonía: $f'(x) = 4x^3 - 3x^2 - 2x + 1$

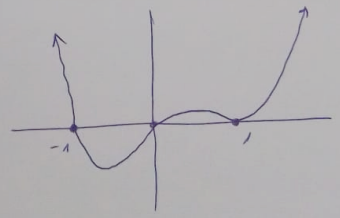
$f' = 0 \xrightarrow{\text{Ruffini}} x = 1 \xrightarrow{2^\circ\text{grado}} x = -0{,}6,\ 0{,}4$

f'	−	+	−	+
f	↘	↗	↘	↗
	−0,6	0,4	1	

- Curvatura $f''(x) = 12x^2 - 6x - 2$

$x = -0{,}23,\ 0{,}73$

	+	−	+	
	∪	∩	∪	
	−0,23	0,73		



Corona Gráficas 3

$f1(x) = \frac{x^2-5x+7}{x-2}$ Dom $f = \mathbb{R} - \{2\}$

- **Cortes ejes:** $x=0 \quad y=-\frac{7}{2}$

  $y=0 \rightarrow x^2-5x+7=0 \rightarrow x=\nexists$

- **Asíntotas**

  Horiz: $\lim_{x\to+\infty} f = +\infty$ $\lim_{x\to-\infty} f = -\infty$ (NO)

  Verti: $\lim_{x\to 2} f = \left(\frac{k}{0}\right) = \pm\infty$ (Sí) $\boxed{x=2}$

  Oblicua: $\frac{f(x)}{x} = \frac{x^2-5x+7}{x^2-2x}$ $\lim_{x\to\pm\infty} \frac{f(x)}{x} = 1$

  $\boxed{m=1}$

  $f(x)-1x = \frac{x^2-5x+7-x^2+2x}{x-2} = \frac{-3x+7}{x-2}$

  $\lim_{x\to\pm\infty} (f(x)-1x) = -3$ $\boxed{n=-3}$

  Asíntota: $\boxed{y=1x-3}$

- **Monotonía** $f'(x) = \frac{(2x-5)(x-2)-(x^2-5x+7)(1)}{(x-2)^2} =$

  $= \frac{2x^2-4x-5x+10-x^2+5x-7}{(x-2)^2} =$

  $= \frac{x^2-4x+3}{(x-2)^2}$

$f'=0 \rightarrow x^2-4x+3=0 \quad x=<\begin{matrix}1\\3\end{matrix}$

f'	+		−		−		+
f	↗	1	↘	2	↘	3	↗

Máx $x=1, y=-3$

mín $x=3, y=1$

- **Curvatura** $f''(x) = \frac{(2x-4)(x-2)^2-(x^2-4x+3)2(x-2)}{(x-2)^4} =$

  f.c. $\rightarrow = \frac{(x-2)[(2x-4)(x-2)-(x^2-4x+3)2]}{(x-2)^4 \, 3}$

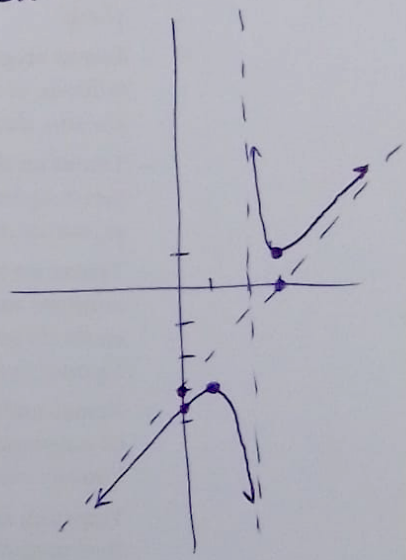
  $= \frac{2x^2-4x-4x+8-2x^2+8x-6}{(x-2)^3} = \frac{x^2-4x+2}{(x-2)^3}$

$f''=0 \rightarrow x^2-4x+2=0 \quad x=2 \quad \nexists$

f''	−		+
f	∩	2	∪

x	Y
0	$-\frac{7}{2} = -3{,}5$
2	$\pm\infty$
$+\infty$	$+\infty$
$-\infty$	$-\infty$
1	−3 M
3	∄ m

x	Y
0	−3
3	0



$f2(x) = \frac{x^3}{x^2+1}$ Dom $f = \mathbb{R}$

- Cortes $x=0$ $y=0$

  $y=0$ $x=0$

- Asíntotas

  Hori $\lim_{x\to+\infty} f = +\infty$ $\lim_{x\to-\infty} f = -\infty$ (no)

  Vert (no)

  Obli $\frac{f(x)}{x} = \frac{x^3}{x^3+x}$; $\lim_{x\to\pm\infty} \frac{f(x)}{x} = 1$ ($m=1$)

  $f(x) - 1x = \frac{x^3 - x^3 - x}{x^2+1} = \frac{-x}{x^2+1}$

  $\lim_{x\to\pm\infty} (f(x) - 1x) = 0$ $n=0$

  Asint. Ob $y = 1x + 0$

- Monotonía $f'(x) = \frac{3x^2(x^2+1) - x^3(2x)}{(x^2+1)^2}$

  $f'(x) = \frac{3x^4 + 3x^2 - 2x^4}{(x^2+1)^2} = \frac{x^4+3x^2}{(x^2+1)^2}$

$f' = 0 \to x^4 + 3x^2 = 0$

$x^2(x^2+3) = 0$ $\begin{cases} x=0 \\ x=\not{3} \end{cases}$

f'	−	0	+
f	↘	0	↗

no Máx

no Mín

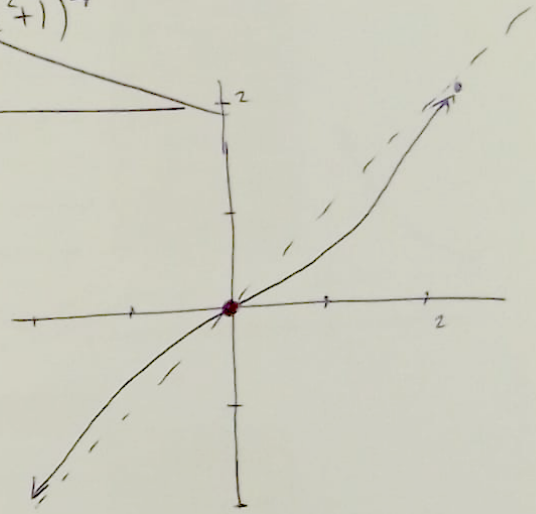
- Curvatura $f''(x) = \frac{(4x^3+6x)(x^2+1)^2 - (x^4+3x^2)}{}$

$f''(x) = \frac{(4x^3+6x)(x^2+1)^2 - (4x^3+6x)2(x^3+1)(2x)}{(x^2+1)^4} =$

f.c. $= \frac{(x^2+1)[(4x^3+6x)}{}$

x	y
0	0
$+\infty$	$+\infty$
$-\infty$	$-\infty$

x	y
0	0
2	2



Corona Gráficas 3

$f3(x) = \dfrac{x^2+1}{x^2-2x}$ $\quad x^2-2x=0$

$x(x-2)=0 \Rightarrow \begin{cases} x=0 \\ x=2 \end{cases}$

$\text{Dom } f = \mathbb{R} - \{0,2\}$

- Cortes $x=0 \quad y=\nexists$

  $y=0 \quad x=\nexists$

- Asíntotas Horiz $\lim_{x\to\pm\infty} f = 1$ (Sí) $\boxed{y=1}$

- Verticales $\lim_{x\to 0} f = \left(\dfrac{1}{0}\right) = \pm\infty$ $\boxed{x=0}$

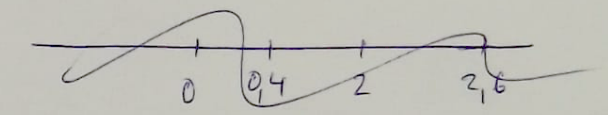
  $\lim_{x\to 2} f = \left(\dfrac{5}{0}\right) = \pm\infty$ $\boxed{x=2}$

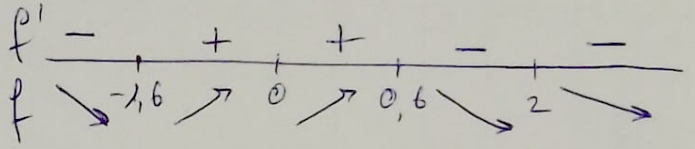
- Monotonía $f' = \dfrac{2x(x^2-2x)-(x^2+1)(2x-2)}{(x^2-2x)^2}$

$$f' = \frac{2x^3-4x^2-2x^3+2x^2-2x+2}{(x^2-2x)^2}$$

$$f' = \frac{-2x^2-2x+2}{(x^2-2x)^2} \; ; \; f'=0 \quad x = \begin{cases} \cancel{2,6} \\ \cancel{0,4} \end{cases}$$

$$x = \begin{cases} -1,6 \\ 0,6 \end{cases}$$

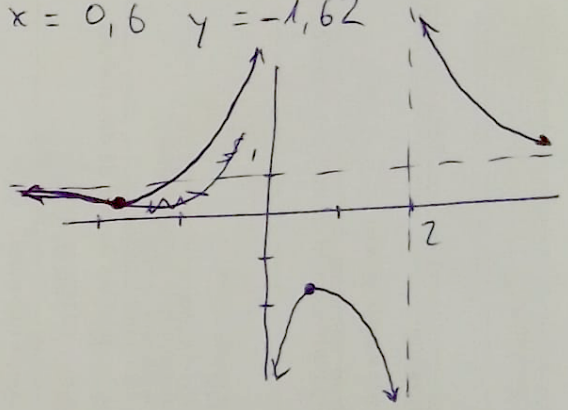




Mínimo $x=-1,6 \quad y=0,62$

Máximo $x=0,6 \quad y=-1,62$

x	y	
$\pm\infty$	1	
0	$\pm\infty$	
2	$\pm\infty$	
−1,6	0,6	m
0,6	−1,6	M



Corona Gráficas 04

① $f(x)=\frac{1}{x^2+1}$ Dom $f=\mathbb{R}$

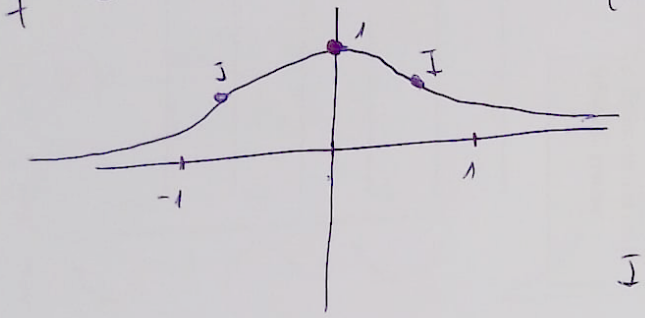
- $(0,1)$
- A.V. $\rightarrow \nexists$
- A.H. $\rightarrow y=0$
- $f'(x)=\frac{-2x}{(x^2+1)^2}$ $\quad f'=0 \rightarrow x=0$

f	↗		↘
f'	+	0	−
		Máx	$(0,1)$

- $f''(x)=\frac{6x^4+4x^2-2}{(x^4+2x^2+1)^2}$ $\quad f''=0$ bicuadrado $x=\begin{cases}\pm 0{,}6\\ \nexists\end{cases}$

f''	+	−	+	
f	∪	$-0{,}6$ ∩	$0{,}6$	∪

P. Inflex $(\pm 0{,}6;\ 0{,}75)$



x	y
M: 0	1
$\pm\infty$	0
I: $\pm 0{,}6$	0,75

② $f(x)=\frac{x}{x^2+1}$ Dom $f=\mathbb{R}$

- $(0,0)$
- A.V. $\rightarrow \nexists$
- A.H. $\rightarrow y=0$
- $f'(x)=\frac{-x^2+1}{x^4+2x^2+1}=\frac{-x^2+1}{(x^2+1)^2}$ $\quad f'=0 \rightarrow x=\pm 1$

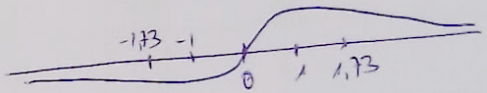
f'	−	+	−	
f	↘	-1 ↗	1	↘

- $f''(x)=\frac{2x^5-4x^3-6x}{(x^4+2x^2+1)^2}=\frac{x(2x^4-4x^2-6)}{(\quad)^2}$

$f''=0 \rightarrow$ f.c. $\rightarrow x=0$
bicuadrada $\rightarrow x=\begin{cases}\pm 1{,}73\\ \nexists\end{cases}$

f''	−	+	−	+	
f	∩	$-1{,}7$ ∪	0	∩ $1{,}7$	∪

x	y	
0	0	I
$\pm\infty$	0	
-1	$-0{,}5$	m
$+1$	$+0{,}5$	M
$-1{,}7$	$+0{,}4$	J
$+1{,}7$	$+0{,}4$	I



Corona Gráficas 04

$f3(x) = \dfrac{1}{x^2-1}$

$\text{Dom } f = \mathbb{R} - \{\pm 1\}$

- $(0,-1)$
- A.V. $x = \pm 1$
- A.H. $y = 0$

- $f'(x) = \dfrac{-2x}{(x^2-1)^2}$

$f' = 0 \quad x = 0$

f'	+	+	0	−	−
f	↗	↗	Máx	↘	↘
			$(0,-1)$		

- $f''(x) = \dfrac{-2x^5+8x^4+4x^3-8x^2-2x}{(x^4-2x^2+1)^2}$

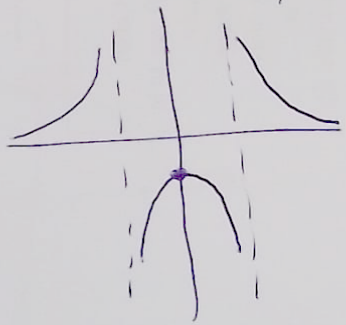
$f'' = 0 \rightarrow$ f.c. $\rightarrow x = 0$

$\rightarrow$ Ruffini $\rightarrow x = \begin{cases} \pm 1 \\ 2^\circ \text{ grado}: \begin{cases} 4{,}24 \\ -0{,}24 \end{cases} \end{cases}$

- $f''(x) = \dfrac{6x^4-4x^2-2}{(x^4-2x^2+1)^2}$

$f'' = 0 \rightarrow x = \begin{cases} \pm 1 \\ \not\exists \end{cases}$

f''	+	−	−	+
f	∪	∩	∩	∪
		-1	1	



$f4(x) = \dfrac{x}{x^2-1}$

$\text{Dom } f = \mathbb{R} - \{\pm 1\}$

- $(0,0)$
- A.V. $x = \pm 1$
- A.H. $y = 0$

- $f'(x) = \dfrac{-x^2-1}{(x^2-1)^2}$

$f' = 0 \rightarrow x = \not\exists$

f'	−	−	−
f	↘	↘	↘
	-1	$+1$	

- $f''(x) = \dfrac{2x^5+4x^3-6x}{(x^4-2x^2+1)^2}$

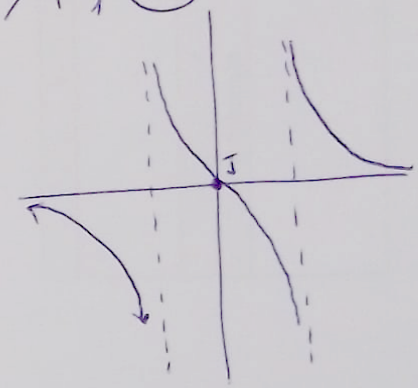
$f'' = 0 \rightarrow$ f.c. $\rightarrow x = 0$

$\rightarrow$ bicuadrada $x = \begin{cases} \pm 1 \\ \not\exists \end{cases}$

f''	−	+	−	+	
f	∩	∪	∩	∪	
	-1	0	1		

P. Inflx: $x =$ $y =$

x	y	
0	0	I
± 1	$\pm \infty$	
$\pm \infty$	0	



Corona gráficas 04

$f5(x) = \frac{1}{x^3+1}$ $\quad Dom f = \mathbb{R} - \{-1\}$

- $(0,1)$
- A.V. $x = -1$
- A.H. $y = 0$

- $f'(x) = \frac{-3x^2}{(x^3+1)^2}$

$f' = 0 \rightarrow x = 0$

f'	$-$		$-$		$-$
f	$\searrow$	-1	$\searrow$	0	$\searrow$

- $f''(x) = \frac{12x^7 + 6x^4 - 6x}{(x^6+2x^3+1)^2}$

$f'' = 0 \rightarrow$ f.c. $\begin{cases} x = 0 \\ 12x^6 + 6x^3 - 6 = 0 \text{ (bicúbica)} \end{cases}$

$t = x^3$

$12t^2 + 6t - 6 = 0$

$t = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$ $\quad x = \begin{cases} \sqrt[3]{-1} = -1 \\ \sqrt[3]{\frac{1}{2}} = 0{,}8 \end{cases}$

f''	$-$		$+$		$-$		$+$
f	$\cap$	-1	$\cup$	0	$\cap$	$0{,}8$	$\cup$

x	y	
0	1	I
$\pm\infty$	0	
-1	$\pm\infty$	
$0{,}8$	$0{,}7$