Ficha 7.6

① $f(x)=\frac{-2}{x-4}$ • $x=-2$ $f(-2)=\frac{-2}{-2-4}=\frac{1}{3}$

$f'(x)=\frac{+2}{(x-4)^2}$; $f'(-2)=\frac{2}{36}=\frac{1}{18}$

$y-\frac{1}{3}=\frac{1}{18}(x+2) \rightarrow y=\frac{1}{18}x+\frac{4}{9}$

• $f'(x)=1 \rightarrow \frac{2}{(x-4)^2}=1 \rightarrow 2=(x-4)^2 \rightarrow$

$\rightarrow x=4\pm\sqrt{2}$

• Igual que antes

② • $f'(x)=\frac{-b(2x-4)}{(x^2-4x)^2}$

$f'(-2)=\frac{-b(-8)}{(12)^2}=4$

$-b(-8)=4(12)^2$

$b=\frac{4\cdot 144}{8}=72$

• $f'(2)=0$ $\frac{-b(0)}{(2^2-4\cdot 2)^2}=0$

$0=0$

Infinitas soluciones. b puede ser cualquier número

② $f(x)=\ln(x^2-4x)$ $f'(x)=\frac{2x-4}{x^2-4x}$

• $f(-2)=\ln 12$ $f'(-2)=\frac{-8}{12}=\frac{-2}{3}$

$y-\ln 12=\frac{-2}{3}(x+2)$

$y=\frac{-2}{3}x-\frac{4}{3}+\ln 12$

• $f'(x)=1$ $\frac{2x-4}{x^2-4x}=1$

$2x-4=x^2-4x$

$x^2-6x+4=0$

$x=3\pm\sqrt{5}$

$y=\begin{cases}1,9\\ \cancel{7}\end{cases}$

Ficha 7.6

• $f'(x)=2$ $\frac{2x-4}{x^2-4x}=2$

$2x-4=2x^2-8x$

$2x^2-10x+4=0$

$x=\begin{cases}\frac{5\pm\sqrt{17}}{2}\end{cases}$

$y=\begin{cases}0,9\\ \cancel{7}\end{cases}$

• $f(x)=\ln(x^2-bx)$ $f'(x)=\frac{2x-b}{x^2-bx}$

$f'(-2)=4$ $\frac{2(-2)-b}{(-2)^2-b(-2)}=4$

$\frac{-4-b}{4+2b}=4$

$-4-b=16+8b$

$-20=9b$

$b=\frac{-20}{9}$

• $f'(2)=0$ $\frac{2\cdot 2-b}{2^2-b\cdot 2}=0$

$4-b=0$

$b=4$

③ $f(x)=-x^2+2$ $f'(-1)=2$ $A(-1,1)$

$f'(x)=-2x$ $f'(1)=-2$ $B(1,1)$

t_A: $y-1=2(x+1) \rightarrow y=2x+3$

t_B: $y-1=-2(x-1) \rightarrow y=-2x+3$

Punto de corte: $2x+3=-2x+3$

$x=0$, $y=3$

④ $f(x)=e^{-x^2+2}$ $f(-1)=e$ $f'(-1)=2e$

$f'(x)=-2xe^{-x^2+2}$ $f(1)=e$ $f'(1)=-2e$

t_A: $y-e=2e(x+1)$ $y=2ex+3e$

t_B: $y-e=-2e(x-1)$ $y=-2ex+3e$

Punto de corte: $2ex+3e=-2ex+3e$

$x=0$, $y=e^2$

Ficha 8.3

Ficha 7.7.

① $f(x) = 3x^5 - 5x^3$ $\quad f'(x) = 15x^4 - 15x^2$

$f'(x) = 0 \rightarrow 15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

$15x^2 = 0 \rightarrow x = 0$

$x^2 - 1 = 0 \rightarrow x = \pm 1$

$\text{Dom} f = \mathbb{R}$

$f''(x) = 60x^3 - 30x$

$f''(0) = 0 \rightarrow$ Pto. Inf/x

$f''(+1) = 30 \rightarrow$ Mínimo

$f''(-1) = -30 \rightarrow$ Máximo

② $f(x) = \sqrt{x^2+1}$ $\quad f'(x) = \frac{2x}{2\sqrt{x^2+1}}$

$f'(x) = 0 \rightarrow x = 0$

$\text{Dom} f = \mathbb{R}$

f'	−	0	+
f	↘		↗

$x = 0 \rightarrow$ Mínimo

Punto $(0, 1)$

Ficha 8.3

③ $f(x) = \frac{x^2+2x+1}{x}$ $\quad f'(x) = \frac{(2x+2)x - (x^2+2x+1)}{x^2}$

$f'(x) = \frac{x^2-1}{x^2}$

$f'(x) = 0 \rightarrow x = \pm 1$ $\quad \text{Dom} f = \mathbb{R} - \{0\}$

f'	+	−	−	+	$x=-1$ Máximo
f	↗	−1	↘ 0 ↘	1	↗ $x=1$ Mínimo

④ $f(x) = 5x^2\sqrt{x+1}$

$f'(x) = 10x\sqrt{x+1} + \frac{5x^2}{2\sqrt{x+1}}$

$f'(x) = \frac{20x(x+1) + 5x^2}{2\sqrt{x+1}} = \frac{20x^2 + 25x^2}{2\sqrt{x+1}}$

$f'(x) = 0 \rightarrow 20x + 25x^2 = 0$

$5x(4 + 5x) = 0$

$x = 0$

$x = -4/5$

$\text{Dom} f = [-1, +\infty)$

f'	+	−		+	$x=-4/5$ Máx.
f	−1 ↗		−4/5 ↘	0 ↗	$x=0$ Mín.

I: −1 0 1

⑤ $f(x) = \begin{cases} 3 - \frac{x^2}{2} & x \leq 2 \\ 6x - x^2 - 7 & x > 2 \end{cases}$

$f'(x) = \begin{cases} -\frac{2x}{2} & x < 2 \\ 6 - 2x & x > 2 \end{cases}$ $\quad f'(x) = 0 \begin{cases} x = 0 \\ x = 3 \end{cases}$

f'	+	−	+	−	$x=0$	Máximo
f	↗	0 ↘	2 ↗	3 ↘	$x=3$	Máximo

Hubiera sido más fácil con la segunda derivada

$f''(x) = \begin{cases} -1 & x < 2 \\ -2 & x > 2 \end{cases}$ $\quad \begin{cases} f''(0) = -1 \rightarrow \text{Máx} \\ f''(3) = -2 \rightarrow \text{Máx.} \end{cases}$

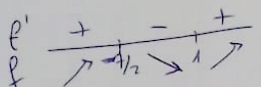
Ficha 7.8.

① $f(x) = e^x(2x^2+x-8)$ $\quad Dom f = \mathbb{R}$

$f'(x) = e^x(2x^2+x-8) + e^x(4x+1) =$

$= e^x(2x^2+5x-7)$

$f'(x) = 0 \rightarrow \begin{cases} e^x = 0 \ \text{A} \\ 2x^2+5x-7 = 0 \rightarrow x = \begin{cases} -7/2 \\ 1 \end{cases} \end{cases}$



$f'(-4) = +$
$f'(0) = -$
$f'(2) = +$

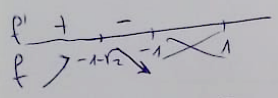
$x = -7/2$ Máx.
$x = 1$ Mín.

② $f(x) = x + \ln(x^2-1)$

$Dom f = \mathbb{R} - [-1,1]$

$f'(x) = 1 + \frac{2x}{x^2-1} = \frac{x^2+2x-1}{x^2-1}$

$f'(x) = 0 \rightarrow x^2+2x-1 = 0 \rightarrow x = \begin{cases} -1-\sqrt{2} \\ -1+\sqrt{2} \ \text{(fora)} \end{cases}$



$f'(-3) = +$
$f'(-2) = -$

$x = -1-\sqrt{2}$ Máx.

③ $f(x) = \text{sen}(2x)$ $\quad Dom f = [0, 2\pi)$

$f'(x) = \cos(2x) \cdot 2$

$f'(x) = 0 \rightarrow \begin{cases} \cos(2x) = 0 \rightarrow 2x = \begin{cases} \frac{\pi}{2} \ (90) \\ \frac{3\pi}{2} \ (270) \end{cases} \\ 2 = 0 \ \text{A} \end{cases}$

$x = \begin{cases} \frac{\pi}{4} \ (45) \\ \frac{3\pi}{4} \ (135) \end{cases}$

$f''(x) = -\text{sen}(2x) \cdot 2 \cdot 2$

$= -4\,\text{sen}(2x)$

$f''\left(\frac{\pi}{4}\right) = -4 \,\text{sen}\, \frac{\pi}{2} = -4 \rightarrow$ Máx

$f''\left(\frac{3\pi}{4}\right) = -4 \,\text{sen}\, \frac{3\pi}{2} = -4(-1) = +4 \rightarrow$ Mín

④ $f(x) = ax^3 + bx$ $\quad f'(x) = 3ax^2 + b$

$(1,1) \rightarrow f(1) = 1 \rightarrow a+b = 1$

$f'(1) = -3 \rightarrow 3a+b = -3$

~~$f(x) = x^3$~~

~~$2a = 2$~~ ~~$a = 1$~~ ~~$b = 0$~~

$2a = -4$

$a = -2 \quad b = 3$

$f(x) = -2x^3 + 3x$

Ficha 7.8

⑤ $f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$f''(x) = 6x + 2a$

Extremo: $f'(2) = 0 \rightarrow 12 + 4a + b = 0$

$T(1,2) \rightarrow 1 + a + b + c = 2$

Inflex $\rightarrow f''(1) = 0 \rightarrow 6 + 2a = 0 \rightarrow a = -3$

$12 + 4(-3) + b = 0 \rightarrow b = 0$

$1 - 3 + c = 2 \rightarrow c = 4$

⑥ $f(x) = x^4 + ax^3 + bx^2 + cx$

$f'(x) = 4x^3 + 3ax^2 + 2bx + c$

$y = x \rightarrow m = 1 \rightarrow f'(0) = 1 \rightarrow 4 + 3a + 2b + c = 1$

$(-1,0) \rightarrow 1 - a + b - c = 0$

Extremo $\rightarrow f'(-1) = 0 \rightarrow -4 + 3a - 2b + c = 0$

$8 + 4b = 1 \rightarrow b = -7/4$

$4 + 3a - \frac{7}{2} + c = 1 \rightarrow 6a + 2c = 1$

$1 - a - \frac{7}{4} - c = 0 \rightarrow -4a - 4c = 3$

$8a = 5 \quad a = 5/8$

$c = -11/8$

- $y = x \rightarrow m = 1 \rightarrow f'(0) = 1 \rightarrow c = 1$
- $(-1,0) \rightarrow 1 - a + b - 1 = 0 \rightarrow -a + b = 0$
- Extremo $f'(-1) = 0 \quad -4 + 3a - 2b + 1 = 0$

$3a - 2b = 3$

$a = b \rightarrow 3a - 2a = 3$

$a = 3 \quad b = 3$