Derivadas 1

$f1'(x) = 9x^2 - \frac{1}{3}$

$f2'(x) =$ (Producto) $(9x^2-1)(x-3x^3)+(3x^3-x)(1-9x^2)$

las sumas hay que hacerlas

$= 9x^3 - 27x^5 - x + 3x^3 + 3x^3 - 27x^5 - x + 9x^3 =$

$= -54x^5 + 24x^3 - 2x$

$f3'(x) =$ Cociente

$\frac{(9x^2-1)(3x-x^3)-(3x^3-x)(3-3x^2)}{(3x-x^3)^2} =$

las sumas y restas se hacen. Las potencias no

$= \frac{27x^3 - 9x^5 - 3x + 3x^3 - 9x^3 + 9x^5 + 3x - 3x^3}{(3x-x^3)^2} =$

$= \frac{16x^3}{(3x-x^3)^2} =$ f.c. $= \frac{16x^3}{x^2(3-x^2)^2} =$

$= \frac{16}{(3-x^2)^2}$

$f_4(x) = 3\left(\frac{x^2}{9}+\frac{x}{6}\right) \rightarrow f_4'(x) = 3\left(\frac{2x}{9}+\frac{1}{6}\right)$

k.u

$f_5' =$ Producto

$3\cdot\left(\frac{x^2}{9}+\frac{x}{6}\right)+3x\left(\frac{2x}{9}+\frac{1}{6}\right) =$

$= \frac{3x^2}{9} + \frac{3x}{6} + \frac{6x^2}{9} + \frac{3x}{6} = \frac{6x^2+9x+12x^2+9x}{18} =$

$= \frac{18x^2+18x}{18} = x^2+x$

$f_6'(x) =$ Potencia $= 4(2x+x^4)^3(2+4x^3) =$

$= (8+16x^3)(2x+x^4)^3$

$f_7'(x) =$ Producto

$1\cdot(x^4+2x)^4 + x\cdot 4(x^4+2x)^3(4x^3+2) =$ f.c.

$= (x^4+2x)^3\left[1(x^4+2x)+x\cdot 4(4x^3+2)\right] =$

$= (x^4+2x)^3(x^4+2x+16x^4+8x) =$

$= (x^4+2x)^3(17x^4+10x)$

Derivadas 1

$f_8'(x) = \frac{1(2x+x^4)^4 - x\cdot 4(2x+x^4)^3(2+4x^3)}{(2x+x^4)^8} =$ f.c.

$= \frac{\cancel{(2x+x^4)^3}\left[1(2x+x^4)-x\cdot 4(2+4x^3)\right]}{(2x+x^4)^{\cancel{8}\,5}} =$

$= \frac{2x+x^4-8x-16x^4}{(2x+x^4)^5} = \frac{-15x^4-6x}{(2x+x^4)^5} =$ f.c $=$

$= \frac{x(-15x^3-6)}{x^5(2+x^3)^5} = \frac{-15x^3-6}{x^4(2+x^3)^5}$

## Derivadas 2

$$f_1(x) = \sqrt{x^4+4x} \rightarrow f_1'(x) = \frac{4x^3+4}{2\sqrt{x^4+4x}} = \frac{2x^3+2}{\sqrt{x^4+4x}}$$

$$f_2(x) = e^{x^4+4x} \rightarrow f_2'(x) = e^{x^4+4x} \cdot (4x^3+4)$$

$$f_3(x) = \ln(x^4+4x) \rightarrow f_3'(x) = \frac{4x^3+4}{x^4+4x}$$

$$g_1(x) = (x+1)\sqrt{x^4+4x} \rightarrow g_1'(x) = 1\cdot\sqrt{x^4+4x} + (x+1)\frac{2x^3+2}{\sqrt{x^4+4x}} =$$

$$= \frac{\sqrt{x^4+4x}^2 + (x+1)(2x^3+2)}{\sqrt{x^4+4x}} = \frac{x^4+4x+2x^4+2x+2x^3+2}{\sqrt{x^4+4x}} =$$

$$= \frac{3x^4+2x^3+6x+2}{\sqrt{x^4+4x}}$$

$$g_2(x) = (x+1)e^{x^4+4x} = 1\cdot e^{x^4+4x} + (x+1)e^{x^4+4x}(4x^3+4) =$$

$$= e^{x^4+4x}\left(1+(x+1)(4x^3+4)\right) =$$

$$= e^{x^4+4x}(1+4x^4+4x+4x^3+4) =$$

$$= e^{x^4+4x}(4x^4+4x^3+4x+5)$$

## Derivadas 2

$$g_3(x) = (x+1)\ln(x^4+4x)$$

$$g_3'(x) = 1\cdot\ln(x^4+4x) + (x+1)\frac{4x^3+4}{x^4+4x} =$$

$$= \ln(x^4+4x) + \frac{4x^4+4x+4x^3+4}{x^4+4x} =$$

$$= \ln(x^4+4x) + \frac{4x^4+4x^3+4x+4}{x^4+4x}$$

$$h_1(x) = \sqrt{\frac{x^4+4x}{x}} \rightarrow h_1'(x) = \frac{\frac{(4x^3+4)x-(x^4+4x)}{x^2}}{2\sqrt{\frac{x^4+4x}{x}}} =$$

$$= \frac{\frac{3x^4}{x^2}}{2\sqrt{\frac{x^4+4x}{x}}} = \frac{3x^2}{2\sqrt{x^3+4}}$$

$$h_2(x) = e^{\frac{x^4+4x}{x}} \qquad h_2'(x) = e^{\frac{x^4+4x}{x}} \cdot \frac{3x^4}{x^2} =$$

$$= e^{\frac{x^4+4x}{x}} \cdot 3x^2$$

$$h_3(x) = \ln\left(\frac{x^4+4x}{x}\right) \qquad h_3'(x) = \frac{\frac{3x^4}{x^2}}{\frac{x^4+4x}{x}} =$$

$$= \frac{3x^2}{x^3+4}$$

Derivadas 3

9a) $f(x)=\frac{x^3}{3}+7x^2-4x \rightarrow f'=\frac{3x^2}{3}+14x-4=$

$=x^2+14x-4$

b) $f(x)=\underbrace{3\cos(2x+\pi)}_{k\cdot u} \rightarrow f'= 3(-\operatorname{sen}(2x+\pi)\cdot 2)=$ $\longrightarrow k\cdot u'$

$f'=-6\operatorname{sen}(2x+\pi)$

c) $f(x)=\frac{1}{3x}+\sqrt{x} \rightarrow f'=\frac{-3}{9x^2}+\frac{1}{2\sqrt{x}}=\frac{-1}{3x^2}+\frac{1}{2\sqrt{x}}=$

$\frac{k}{u}$

$=\frac{-2\sqrt{x}+3x^2}{6x^2\sqrt{x}}$

d) $f(x)=\frac{x^2}{x+1} \rightarrow f'=\frac{2x(x+1)-x^2}{(x+1)^2}=\frac{x^2+2x}{(x+1)^2}=$ /

10a) $f(x)=(5x-2)^3 \rightarrow f'(x)=3(5x-2)^2\,5=$

$=15(5x-2)^2$

b) $f(x)=\left(\frac{1}{3x}+\frac{x}{3}\right)^4 \rightarrow f'(x)=4\left(\frac{1}{3x}+\frac{x}{3}\right)^3\left(\frac{-3}{9x^2}+\frac{1}{3}\right)=$

$=4\left(\frac{1}{3x}+\frac{x}{3}\right)^3\left(\frac{-1}{3x^2}+\frac{1}{3}\right)$

Derivadas 3

10c) $\frac{2(6-x)^1(-1)}{3\sqrt[3]{(6-x)^4}}=\frac{-2(6-x)}{3(6-x)\sqrt[3]{(6-x)}}=$

$=\frac{-2}{3\sqrt[3]{6-x}}$

d) $f(x)=\frac{e^x+e^{-x}}{e^x}=\frac{(e^x+e^{-x}(-1))e^x-(e^x+e^{-x})e^x}{(e^x)^2}=$

f.c. $=\frac{e^x(e^x-e^{-x}-e^x-e^{-x})}{(e^x)^2}=\frac{-2e^{-x}}{e^x}=\frac{-2}{(e^x)^2}=$

$=\frac{-2}{e^{2x}}$

11 a) $f(x)=\sqrt{\operatorname{arc\,cos} e^x}$

$f'(x)=\frac{\frac{-1\cdot e^x}{\sqrt{1-(e^x)^2}}}{2\sqrt{\operatorname{arc\,cos} e^x}}=\frac{-e^x}{2\sqrt{\operatorname{arc\,cos} e^x}\sqrt{1-e^{2x}}}$

11 b) $f(x)=\log(\operatorname{sen} x^2)$

$f'=\frac{\cos x^2\cdot 2x}{\operatorname{sen} x^2\cdot \ln 10}=\frac{2x}{\operatorname{tg} x^2\cdot \ln 10}$

Derivadas 3

11 c) $f(x)=\operatorname{sen}^2 x+e^{\cos x}$

$f'(x)=2\operatorname{sen} x\cdot\cos x+e^{\cos x}\cdot(-\operatorname{sen} x)$

f.c. $=\operatorname{sen} x(2\cos x - e^{\cos x})$

~~11 d) $f(x)=\frac{\sqrt[4]{2x}}{2^{x-1}}$~~

~~$f'(x)=\frac{\frac{2x}{4\sqrt[4]{(2x)^3}}}{2^{x-1}\cdot 1\cdot \ln 2}=$~~

~~$=\frac{x}{\sqrt[4]{8x^3}\cdot 2^{x-1}\cdot \ln 2}$~~

11d) $f(x)=\frac{\sqrt[4]{2x}}{2^{x-1}}$ (cociente)

$f'(x)=\frac{\frac{2x}{4\sqrt[4]{(2x)^3}}\cdot 2^{x-1}-\sqrt[4]{2x}\cdot 2^{x-1}\cdot 1\cdot \ln 2}{(2^{x-1})^2}=$

$=$ ______

$=\frac{\frac{x}{2\sqrt[4]{8x^3}}\cdot 2^{x-1}-\sqrt[4]{2x}\cdot 2^{x-1}\cdot \ln 2}{(2^{x-1})^2}=$ f.c.

$=\frac{2^{x-1}\left[\frac{x}{2\sqrt[4]{8x^3}}-\sqrt[4]{2x}\cdot\ln 2\right]}{(2^{x-1})^2}=$

$=\frac{\frac{x-\sqrt[4]{16x^4}\,\ln 2}{2\sqrt[4]{8x^3}}}{2^{x-1}}=\frac{x-2x\ln 2}{2\sqrt[4]{8x^3}\,2^{x-1}}=$

$=\frac{x(1-2\ln 2)}{2^x\cdot\sqrt[4]{8x^3}}$

Derivadas 4

9e) $f(x)=\frac{1}{7x+1}+\frac{\sqrt{2x}}{3}$

$$f'(x)=\frac{-7}{(7x+1)^2}+\frac{\frac{\cancel{2}}{\cancel{2}\sqrt{x}}}{3}=\frac{-7}{(7x+1)^2}+\frac{1}{3\sqrt{x}}$$

f) $f(x)=x\,\text{sen}\frac{x}{2}$

$$f'(x)=\text{sen}\frac{x}{2}+x\cos\frac{x}{2}\cdot\frac{1}{2}=\text{sen}\frac{x}{2}+\frac{x}{2}\cos\frac{x}{2}$$

g) $f(x)=\frac{1}{\sqrt{x-4}}$ ; $f'(x)=\frac{-\frac{1}{2\sqrt{x-4}}}{(\sqrt{x-4})^2}=-\frac{1}{2\sqrt{x-4}(x-4)}$

h) $f(x)=\ln 3x+e^{-x}$; $f'(x)=\frac{1}{\cancel{3}x}\cdot\cancel{3}+e^{-x}(-1)=$

$$=\frac{1}{x}-e^{-x}$$

i) $f(x)=\frac{tg\,x}{2}$ ; $f'=\frac{\frac{1}{\cos^2 x}}{2}=\frac{1}{2\cos^2 x}$

j) $f(x)=\sqrt{3}\,\text{arc sen}\,2x$; $f'(x)=\sqrt{3}\,\frac{2}{\sqrt{1-4x^2}}=$

$$=\frac{2\sqrt{3}}{\sqrt{1-4x^2}}$$

Derivadas 4

10e) $f(x)=\sqrt{\frac{x^3}{x^2-4}}$

$$f'=\frac{\frac{3x^2(x^2-4)-x^3(2x)}{(x^2-4)^2}}{2\sqrt{\frac{x^3}{x^2-4}}}=\frac{\frac{x^4-12x^2}{(x^2-4)^2}}{2\sqrt{\frac{x^3}{x^2-4}}}=$$

$$=\frac{x^4-12x^2}{2(x^2-4)^2\sqrt{\frac{x^3}{x^2-4}}}$$

f) $f(x)=\left(\frac{x}{2}\right)^3\cdot e^{2x+1}$

$$f'(x)=3\left(\frac{x}{2}\right)^2\cdot\frac{1}{2}\cdot e^{2x+1}+\left(\frac{x}{2}\right)^3 e^{2x+1}\cdot 2=$$

$$=\left(\frac{x}{2}\right)^2 e^{2x+1}\cdot\left(\frac{3}{2}+\frac{x}{\cancel{2}}\cdot\cancel{2}\right)=\left(\frac{x}{2}\right)^2 e^{2x+1}\left(\frac{3}{2}+x\right)$$

g) $f(x)=x^3\cos^2 3x$

$$f'(x)=3x^2\cos^2 3x+x^3\cdot 2\cdot\cos 3x\cdot(-\text{sen}\,3x)\cdot 3=$$

$$=3x^2\cos 3x\left(\cos 3x+2x(-\text{sen}\,3x)\right)=$$

$$=3x^2\cos 3x\left(\cos 3x-2x\,\text{sen}\,3x\right)$$

Derivadas 4

10h) $f(x)=tg^3 x^2$

$$f'(x)=3tg^2x^2\cdot\frac{2x}{\cos^2 x^2}=\frac{6x\,tg^2(x^2)}{\cos^2(x^2)}$$

i) $f(x)=\sqrt{7\ln x}$ ; $f'(x)=\frac{7\frac{1}{x}}{2\sqrt{7\ln x}}=\frac{7}{2x\sqrt{7\ln x}}$

j) $f(x)=\text{arc tg}\frac{x^2}{3}$ ; $f'(x)=\frac{\frac{2x}{3}}{1+\frac{x^4}{9}}=$

$$=\frac{\frac{2x}{\cancel{3}}}{\frac{9+x^4}{\cancel{9}\,3}}=\frac{6x}{9+x^4}$$

11e) $f(x)=e^{\text{sen}x}\cdot\ln tg x$

$$f'(x)=e^{\text{sen}x}\cdot\cos x\cdot\ln tg x+e^{\text{sen}x}\cdot\frac{\frac{1}{\cos^2 x}}{tg x}=$$

$$=e^{\text{sen}x}\left(\cos x\ln tg x+\frac{1}{\cos^2 x\cdot tg x}\right)=$$

$$=e^{\text{sen}x}\left(\cos x\ln tg x+\frac{1}{\cos x\cdot\text{sen}x}\right)$$

Derivadas 4

11f) $f(x)=3\cos(\ln x)$

$$f'(x)=3\left(-\text{sen}(\ln x)\cdot\frac{1}{x}\right)=\frac{-3\,\text{sen}(\ln x)}{x}$$

g) $f(x)=\sqrt{x+\sqrt{x}}$ ; $f'=\frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}=$

$$=\frac{\frac{2\sqrt{x}+1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}=\frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$$

h) $f=\text{arc tg}\frac{1-x}{1+x}$ ; $f'=\frac{\frac{-1(1+x)-(1-x)}{(1+x)^2}}{1+\frac{(1-x)^2}{(1+x)^2}}=$

$$=\frac{\frac{-2}{\cancel{(1+x)^2}}}{\frac{(1+x)^2+(1-x)^2}{\cancel{(1+x)^2}}}=\frac{-2}{(1+x)^2+(1-x)^2}$$

Derivadas 4

11 i) $f(x) = 7^{\sqrt{x}} + \dfrac{\cos x}{x^2}$

$$f'(x) = 7^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \ln 7 + \frac{-\operatorname{sen} x \cdot x^2 - \cos x \cdot 2x}{x^4}$$

$$= 7^{\sqrt{x}} \frac{\ln 7}{2\sqrt{x}} - \frac{\cancel{x}(\operatorname{sen} x \cdot x + \cos x \cdot 2)}{x^{\cancel{4}\,3}} =$$

$$= 7^{\sqrt{x}} \frac{\ln 7}{2\sqrt{x}} - \frac{x \operatorname{sen} x + 2\cos x}{x^3}$$

11 j) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$$f'(x) = \frac{(e^x - e^{-x} \cdot (-1))(e^x + e^{-x}) - (e^x - e^{-x})(e^x + e^{-x}(-1))}{(e^x + e^{-x})^2}$$

$$= \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2} =$$

$$= \frac{\cancel{(}e^{2x} + 1 + 1 + e^{-2x} - e^{2x} + 1 + 1 - e^{-2x}}{(e^x + e^{-x})^2} =$$

$$= \frac{4}{(e^x + e^{-x})^2}$$