Ejercicios de Funciones: Integrales Indefinidas.

1. $\int \frac{1}{x^2\sqrt[5]{x^2}}\,dx$

$$\int \frac{1}{x^2\sqrt[5]{x^2}}\,dx = \int x^{-2}x^{\frac{-2}{5}}\,dx = \int x^{\frac{-12}{5}}\,dx = \frac{x^{\frac{-12}{5}+1}}{\frac{-12}{5}+1} + C =$$

$$= \frac{x^{\frac{-7}{5}}}{\frac{-7}{5}} + C = -\frac{5}{7\sqrt[5]{x^7}} + C$$

2. $\int (x+2)^3\,dx$

$$\int (x+2)^3\,dx = \frac{1}{4}(x+2)^4 + C$$

3. $\int (2x+1)(x^2+x+1)\,dx$

$$\int (2x+1)(x^2+x+1)\,dx = \frac{1}{2}(x^2+x+1)^2 + C$$

4. $\int \frac{x+1}{\sqrt[3]{x^2+2x+7}}\,dx$

$$\int \frac{x+1}{\sqrt[3]{x^2+2x+7}}\,dx = \frac{1}{2}\int (2x+2)(x^2+2x+7)^{\frac{-1}{3}}\,dx =$$

$$= \frac{1}{2}\frac{(x^2+2x+7)^{\frac{2}{3}}}{\frac{2}{3}} + C = \frac{3}{4}\sqrt[3]{(x^2+2x+7)^2} + C$$

5. $\int sen\,2x\,\cos 2x\,dx$

$$\int sen\,2x\,\cos 2x\,dx = \frac{1}{2}\int sen\,2x\,\cos 2x\cdot 2\,dx = \frac{1}{4}sen^2\,2x + C$$

6. $\int \text{sen}^4 x\cos x\,dx =$

$$\int \text{sen}^4 x\cos x\,dx = \frac{1}{5}\text{sen}^5 x + C$$

7. $\int \operatorname{tg}^2 x \sec^2 x \, dx$

$$\int \operatorname{tg}^2 x \sec^2 x \, dx = \frac{1}{3}\operatorname{tg}^3 x + C$$

8. $\int \frac{arc\ tg\ x}{1+x^2} dx$

$$\int \frac{arc\ tg\ x}{1+x^2} dx = \frac{1}{2}(arc\ tg\ x)^2 + C$$

9. $\int \frac{2x}{1+x^2} dx$

$$\int \frac{2x}{1+x^2} dx = \ln\left(1+x^2\right) + C$$

10. $\int \operatorname{tg} x \, dx$

$$\int tg\ x\, dx = \int \frac{sen\ x}{\cos x} dx = -\int \frac{-sen\ x}{\cos x} dx = -\ln \cos x + C$$

11. $\int \frac{5^{3x}}{5^{3x}+7} dx$

$$\int \frac{5^{3x}}{5^{3x}+7} dx = \frac{1}{3}\frac{1}{\ln 5}\int \frac{3 \cdot 5^{3x} \ln 5}{5^{3x}+7} dx = \frac{1}{3\ln 5} \ln\left(5^{3x}+7\right) + C$$

12. $\int \frac{1}{x \ln x} dx$

$$\int \frac{1}{x\ln x} dx = \int \frac{\frac{1}{x}}{\ln x} dx = \ln(\ln x) + C$$

13. $\int \frac{1}{\cos^2 x\, tgx} dx$

$$\int \frac{1}{\cos^2 x\, tgx} dx = \int \frac{\frac{1}{\cos^2 x}}{tgx} dx = \ln(tgx) + C$$

14. $\int \frac{x+1}{x} dx$

$$\int \frac{x+1}{x}dx = \int \left(1+\frac{1}{x}\right)dx = x + \ln x + C$$

15. $\int \frac{x+1}{x-5}dx$

$$\int \frac{x+1}{x-5}dx = \int \frac{x+1-5+5}{x-5}dx = \int \frac{x-5}{x-5}dx + \int \frac{6}{x-5}dx =$$
$$= x + 6\ln(x-5) + C$$

16. $\int \frac{3x^3+5x}{x^2+1}dx$

$$\begin{array}{l|l} 3x^3+5x & x^2+1 \\ \hline \quad 2x & 3x \end{array}$$

$$\int \frac{3x^3+5x}{x^2+1}dx = \int \left(3x + \frac{2x}{x^2+1}\right)dx = \frac{3}{2}x^2 + \ln\left(x^2+1\right) + C$$

17. $\int e^{2x+2}\,dx$

$$\int e^{2x+2}\,dx = \frac{1}{2}e^{2x+2} + C$$

18. $\int 5^x\,dx$

$$\int 5^x\,dx = \frac{5^x}{\ln 5}$$

19. $\int 2^x\,5^x\,dx$

$$\int 2^x\,5^x\,dx = \int 10^x\,dx = \frac{10^x}{\ln 10} + C$$

20. $\int 8^{3x+1}\,dx$

$$\int 8^{3x+1}\,dx = \frac{1}{3}\int 8^{3x+1}\,3\,dx = \frac{1}{3\ln 8}8^{3x+1} + C$$

21. $\int \frac{e^{\ln x}}{x}dx$

$$\int \frac{e^{\ln x}}{x}\,dx = \int \frac{1}{x}\, e^{\ln x}\,dx = e^{\ln x} + C$$

22. $\int \frac{e^{\operatorname{arc\,sen} x}}{\sqrt{1-x^2}}\,dx$

$$\int \frac{e^{\operatorname{arc\,sen} x}}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}}\, e^{\operatorname{arc\,sen} x}\,dx = e^{\operatorname{arc\,sen} x} + C$$

23. $\int (3 - \operatorname{sen} x)\,dx$

$$\int (3 - \operatorname{sen} x)\,dx = 3x + \cos x$$

24. $\int \operatorname{sen}(3x+5)\,dx$

$$\int \operatorname{sen}(3x+5)\,dx = \frac{1}{3}\int \operatorname{sen}(3x+5)\,3\,dx = -\frac{1}{3}\cos(3x+5) + C$$

25. $\int (x+1)\operatorname{sen}(x^2+2x+3)\,dx$

$$\int (x+1)\operatorname{sen}(x^2+2x+3)\,dx = \frac{1}{2}\int (2x+2)\operatorname{sen}(x^2+2x+3)\,dx =$$

$$= -\frac{1}{2}\cos(x^2+2x+3) + C$$

26. $\int e^x \operatorname{sen} e^x\,dx$

$$\int e^x \operatorname{sen} e^x\,dx = -\cos e^x$$

27. $\int \operatorname{sen} 2x\,dx$

$$\int \operatorname{sen} 2x\,dx = \frac{1}{2}\int \operatorname{sen} 2x \cdot 2\,dx = -\frac{1}{2}\cos 2x + C$$

28. $\int \operatorname{sen}^3 x\,dx$

$$\int \operatorname{sen}^3 x\,dx = \int \operatorname{sen}^2 x\ \operatorname{sen} x\,dx = \int (1-\cos^2 x)\operatorname{sen} x\,dx =$$

$$\int (\operatorname{sen} x - \cos^2 x\ \operatorname{sen} x)\,dx = -\cos x + \frac{1}{3}\cos^3 x + C$$

29. $\int \frac{\cos(\ln x)}{x}dx$

$$\int \frac{\cos(\ln x)}{x}dx = \int \cos(\ln x)\frac{1}{x}dx = \operatorname{sen}(\ln x) + C$$

30. $\int \frac{5}{\cos^2 x}dx$

$$\int \frac{5}{\cos^2 x}dx = 5\,tgx + C$$

31. $\int (3+3\operatorname{tg}^2 x)dx$

$$\int (3+3tg^2 x)dx = 3\int (1+tg^2 x)dx = 3tg\,x + C$$

32. $\int \sec^2(5x+3)dx$

$$\int \sec^2(5x+3)dx = \frac{1}{5}\int \sec^2(5x+3)5\,dx = \frac{1}{5}tg(5x+3) + C$$

33. $\int \operatorname{tg}^2 x\ dx$

$$\int \operatorname{tg}^2 x\ dx = \int (1+\operatorname{tg}^2 x - 1)dx = \int (1+\operatorname{tg}^2 x)dx - \int dx = \operatorname{tg} x - x + C$$

34. $\int \frac{2x+5}{\sqrt{9-x^2}}dx$

$$\int \frac{2x+5}{\sqrt{9-x^2}}dx = \int \frac{2x}{\sqrt{9-x^2}}dx + \int \frac{5}{\sqrt{9-x^2}}dx$$

$$= -\int (9-x^2)^{-\frac{1}{2}}(-2x)dx + \frac{5}{3}\cdot 3\int \frac{\frac{1}{3}}{\sqrt{1-\left(\frac{x}{3}\right)^2}}dx =$$

$$= -\frac{(9-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + 5\operatorname{arc\,sen}\left(\frac{x}{3}\right) + C = -2\sqrt{9-x^2} + 5\operatorname{arc\,sen}\left(\frac{x}{3}\right) + C$$

35. $\int \frac{2^x}{\sqrt{1-4^x}}dx$

$$\int \frac{2^x}{\sqrt{1-4^x}}dx = \int \frac{2^x}{\sqrt{1-\left(2^x\right)^2}}dx = \frac{1}{\ln 2}\int \frac{2^x \ln 2}{\sqrt{1-\left(2^x\right)^2}}dx =$$

$$= \frac{1}{\ln 2}\text{arc sen}\left(2^x\right) + C$$

36. $\int \frac{x}{\sqrt{9-2x^4}}dx$

$$\int \frac{x}{\sqrt{9-2x^4}}dx = \int \frac{x}{\sqrt{9\left[1-\left(\frac{\sqrt{2}\,x^2}{3}\right)^2\right]}}dx =$$

$$= \frac{1}{3}\frac{3}{2\sqrt{2}}\int \frac{\frac{2\sqrt{2}}{3}x}{\sqrt{1-\left(\frac{\sqrt{2}\,x^2}{3}\right)^2}}dx = \frac{1}{2\sqrt{2}}\text{arc sen}\left(\frac{\sqrt{2}}{3}x^2\right) + C$$

37. $\int \cos x \sqrt{sen\, x}\, dx$

$$\int \cos x \sqrt{sen\, x}\, dx = \int \left(sen\, x\right)^{\frac{1}{2}} \cos x\, dx = \frac{\left(sen\, x\right)^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= \frac{2}{3} sen\, x \sqrt{sen\, x} + C$$

38. $\int \frac{dx}{x\left(1+\ln x\right)^3}$

$$\int \frac{dx}{x\left(1+\ln x\right)^3} = \int \left(1+\ln x\right)^{-3}\frac{1}{x}dx = -\frac{\left(1+\ln x\right)^{-2}}{2} + C =$$

$$= -\frac{1}{2\left(1+\ln x\right)^2} + C$$

39. Escribe la **función primitiva** de y = x² + 2x cuya representación gráfica pasa por él punto (1, 3).

$$\int \left(x^2+2x\right)dx = \frac{x^3}{3}+\frac{2x^2}{2}+C = \frac{x^3}{3}+x^2+C \;;\; y = \frac{x^3}{3}+x^2+C$$

$$3 = \frac{1^3}{3}+1^2+C \qquad C = \frac{5}{3} \;;\; y = \frac{x^3}{3}+x^2+\frac{5}{3}$$

40. Calcular la ecuación de la curva que pasa por P(1, 5) y cuya pendiente en cualquier punto es 3x² + 5x − 2.

$$y' = 3x^2 + 5x - 2 \;;\; y = \int \left(3x^2 + 5x - 2\right)dx = x^3 + \frac{5}{2}x^2 - 2x + C$$

$$5 = 1 + \frac{5}{2} - 2 + C \qquad C = \frac{7}{2} \;;\; y = x^3 + \frac{5}{2}x^2 - 2x + \frac{7}{2}$$

41. Hallar la **primitiva de la función** $f(x) = x\sqrt{x^2 - 1}$, que se anula para x = 2

$$\int x\sqrt{x^2-1}\,dx = \frac{1}{2}\int 2x\left(x^2-1\right)^{\frac{1}{2}}dx = \frac{1}{2}\frac{\left(x^2-1\right)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\frac{1}{3}\sqrt{\left(2^2-1\right)^3} + C = 0 \qquad C = -\sqrt{3}$$

$$F(x) = \frac{1}{3}\sqrt{\left(x^2-1\right)^3} - \sqrt{3}$$

42. $\int x \operatorname{sen} x\, dx$

$$u = x \xrightarrow{\text{derivar}} u' = 1$$

$$v' = \operatorname{sen} x \xrightarrow{\text{integrar}} v = -\cos x$$

$$\int x \operatorname{sen} x\, dx = -x\cos x + \int \cos x\, dx = = -x\cos x + \operatorname{sen} x + C$$

43. $\int \frac{\ln x}{x^3}dx$

$$u = \ln x \xrightarrow{\text{derivar}} u' = \frac{1}{x}$$

$$v' = \frac{1}{x^3} \xrightarrow{\text{integrar}} v = -\frac{1}{2x^2}$$

$$\int \frac{\ln x}{x^3}dx = -\frac{1}{2x^2}\ln x + \frac{1}{2}\int \frac{1}{x^3}dx = -\frac{1}{2x^2}\ln x - \frac{1}{4x^2} + C$$

44. $\int \left(x^3 + 5x^2 - 2\right)e^{2x}dx$

$$u = x^3 + 5x^2 - 2 \xrightarrow{\text{derivar}} u' = 3x^2 + 10x$$

$$v' = e^{2x} \xrightarrow{\text{integrar}} v = \frac{1}{2}e^{2x}$$

$$\int \left(x^3 + 5x^2 - 2\right)e^{2x}dx = \frac{1}{2}\left(x^3 + 5x^2 - 2\right)e^{2x} - \frac{1}{2}\int \left(3x^2 + 10x\right)e^{2x}dx =$$

$$u = 3x^2 + 10x \xrightarrow{\text{derivar}} u' = 6x + 10$$

$$v' = e^{2x} \xrightarrow{\text{integrar}} v = \frac{1}{2}e^{2x}$$

$$= \frac{1}{2}\left(x^3+5x^2-2\right)e^{2x} - \frac{1}{2}\left(\frac{1}{2}\left(3x^2+10x\right)e^{2x} - \frac{1}{2}\int\left(6x+10\right)e^{2x}\,dx\right) =$$

$u = 6x+10 \xrightarrow{\text{derivar}} u' = 6$

$v' = e^{2x} \xrightarrow{\text{integrar}} v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}\left(x^3+5x^2-2\right)e^{2x} - \frac{1}{4}\left(3x^2+10x\right)e^{2x} + \frac{1}{8}\left(6x+10\right)e^{2x} + \frac{3}{4}\int e^{2x}dx =$$

$$= \frac{1}{2}\left(x^3+5x^2-2\right)e^{2x} - \frac{1}{4}\left(3x^2+10x\right)e^{2x} + \frac{1}{8}\left(6x+10\right)e^{2x} - \frac{3}{8}e^{2x} + C =$$

$$= \left(\frac{1}{2}x^3+\frac{7}{4}x^2-\frac{7}{4}x-\frac{1}{8}\right)e^{2x} + C$$

45. $\int \frac{3x^2-2x+5}{(x+3)^3}dx$

$$\frac{3x^2-2x+5}{(x+3)^3} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3}$$

$$3x^2-2x+5 = A(x+3)^2 + B(x+3) + C$$

$$\int \frac{3x^2-2x+5}{(x+3)^3}dx = \int\frac{3}{x+3}dx - \int\frac{20}{(x+3)^2}dx + \int\frac{38}{(x+3)^3}dx =$$

$$= 3\ln(x+3) + \frac{20}{x+3} - \frac{19}{(x+3)^2} + C$$

46. $\int x\sqrt{1+x}\,dx$

$\sqrt{1+x}=t \;;\; 1+x=t^2 \qquad x=t^2-1 \;;\; dx = 2t\,dt$

$$\int\left(t^2-1\right)\cdot t\cdot 2t\,dt = \int\left(2t^4-2t^2\right)dt = \frac{2}{5}t^5 - \frac{2}{3}t^3 + C$$

$$\frac{2}{5}\left(\sqrt{1+x}\right)^5 - \frac{2}{3}\left(\sqrt{1+x}\right)^3 + C =$$

$$= \frac{2}{5}(1+x)^2\sqrt{1+x} - \frac{2}{3}(1+x)\sqrt{1+x} + C$$

47. $\int \frac{dx}{\sqrt{1+e^x}}$

$t = \sqrt{1+e^x} \;;\; 1+e^x = t^2 \qquad e^x = t^2-1$

$e^x\,dx = 2t\,dt \qquad dx = \frac{2t\,dt}{t^2-1}$

$$\int\frac{2t\,dt}{\left(t^2-1\right)t} = 2\int\frac{dt}{t^2-1}$$

$$\frac{1}{t^2-1}=\frac{A}{t+1}+\frac{B}{t-1} \qquad 1=A(t-1)+B(t+1)$$

$$t=-1 \qquad 1=-2A \qquad A=-\frac{1}{2}$$

$$t=1 \qquad 1=2B \qquad B=\frac{1}{2}$$

$$2\int\frac{dt}{t^2-1}=-\int\frac{dt}{t+1}+\int\frac{dt}{t-1}=-\ln(t+1)+\ln(t-1)+C=\ln\left(\frac{t-1}{t+1}\right)+C$$

$$\int\frac{dx}{\sqrt{1+e^x}}=\ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right)+C$$

48. $\int\frac{e^{4x}+3}{e^{3x}}dx$

$$e^x=t \;;\; e^x dx=dt \qquad dx=\frac{dt}{t}$$

$$\int\frac{e^{4x}+3}{e^{3x}}dx=\int\frac{t^4+3}{t^3\cdot t}dt=\int\frac{t^4+3}{t^4}dt=\int dt+3\int\frac{dt}{t^4}=t-\frac{1}{t^3}+C=$$

$$=e^x-\frac{1}{e^{3x}}+C$$

49. $\int arc\ tg\ x\,dx$

$$u=arc\ tg\ x \xrightarrow{derivar} u'=\frac{1}{1+x^2}$$

$$v'=1 \xrightarrow{integrar} v=x$$

$$\int arc\ tg\ x\,dx=x\,arc\ tg\ x-\int\frac{x}{1+x^2}dx=$$

$$=x\,arc\ tg\ x-\frac{1}{2}\ln\left(1+x^2\right)+C$$

50. $\int\frac{2x^2+5x-1}{x^3+x^2-2x}dx$

$$\frac{2x^2+5x-1}{x^3+x^2-2x}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+2}$$

Se efectúa la suma:

$$\frac{2x^2+5x-1}{x^3+x^2-2x}=\frac{A(x-1)(x+2)+Bx(x+2)+Cx(x-1)}{x(x-1)(x+2)}$$

Como las dos fracciones tienen el mismo denominador, los numeradores han de ser iguales:

$$2x^2+5x-1=A(x-1)(x+2)+Bx(x+2)+Cx(x-1)$$

Calculamos los coeficientes de A, B y C dando a la x los valores que anulan al denominador.

$x = 0 \qquad -1 = A(-1)(2) \qquad A = \frac{1}{2}$

$x = 1 \qquad 6 = B(1)(3) \qquad B = 2$

$x = -2 \qquad -3 = C(-2)(-3) \qquad C = -\frac{1}{2}$

Se calculan las integrales de las fracciones simples:

$$\int \frac{2x^2+5x-1}{x^3+x^2-2x}dx = \frac{1}{2}\int\frac{dx}{x} + 2\int\frac{dx}{x-1} - \frac{1}{2}\int\frac{dx}{x+2} =$$

$$= \frac{1}{2}\ln(x) + 2\ln(x-1) - \frac{1}{2}\ln(x+2) + C$$

51. $\int \frac{x^2+1}{(x+1)^2(x-3)}dx$

$$\frac{x^2+1}{(x+1)^2(x-3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3}$$

$$\frac{x^2+1}{(x+1)^2(x-3)} = \frac{A(x+1)(x-3)+B(x-3)+C(x+1)^2}{(x+1)^2(x-3)}$$

$$x^2+1 = A(x+1)(x-3)+B(x-3)+C(x+1)^2$$

Para calcular los valores de A, B y C, damos a x los valores que anulan al denominador y otro más.

$x = -1 \qquad 2 = -4B \qquad B = -\frac{1}{2}$

$x = 3 \qquad 10 = 16C \qquad C = \frac{5}{8}$

$x = 0 \qquad 1 = -3A - 3B + C \qquad A = \frac{3}{8}$

$$\int \frac{x^2+1}{(x+1)^2(x-3)}dx = \frac{3}{8}\int\frac{dx}{x+1} - \frac{1}{2}\int\frac{dx}{(x+1)^2} + \frac{5}{8}\int\frac{dx}{x-3} =$$

$$= \frac{3}{8}\ln(x+1) - \frac{1}{2(x+1)} + \frac{5}{8}\ln(x-3) + C$$

52. $\int \frac{x^3\sqrt{1+x^4}}{\sqrt{1+x^4}+1}dx$

$\sqrt{1+x^4} = t \quad ; 1+x^4 = t^2$

$4x^3\,dx = 2t\,dt \qquad dx = \frac{t}{2x^3}dt$

$$\int \frac{x^3 t}{t+1}\cdot\frac{t}{2x^3}dt = \frac{1}{2}\int\frac{t^2}{t+1}dt =$$

$$\begin{array}{r|l} t^2 & t+1 \\ \hline -t^2-t & t-1 \\ -t & \\ t+1 & \\ \hline 1 & \end{array}$$

$$=\frac{1}{2}\left(t-1+\frac{1}{t+1}\right)dt=\frac{1}{4}t^2-\frac{1}{2}t+\frac{1}{2}\ln(t+1)+C=$$

$$=\frac{1}{4}(1+x^4)-\frac{1}{2}\sqrt{1+x^4}+\frac{1}{2}\ln\left(\sqrt{1+x^4}+1\right)+C$$

53. $\int \frac{1+e^x}{1-e^x}dx$

$e^x=t$; $e^x dx=dt$ $\qquad dx=\frac{dt}{t}$

$$\int\frac{1+t}{1-t}\frac{dt}{t}=\int\frac{-1-t}{t(t-1)}dt$$

$$\frac{-1-t}{t(t-1)}=\frac{A}{t}+\frac{B}{t-1}\ ;\ -1-t=A(t-1)+Bt$$

$t=1 \qquad -2=B$

$t=0 \qquad -1=-A \qquad A=1$

$$\int\frac{dt}{t}-2\int\frac{dt}{t-1}=\ln t-2\ln(t-1)+C=$$

$$=\ln e^x-2\ln(e^x-1)+C=x-\ln(e^x-1)^2+C$$

54. $\int x^2\ln x\,dx$

$u=\ln x \xrightarrow{\text{derivar}} u'=\frac{1}{x} \qquad v'=x^2 \xrightarrow{\text{integrar}} v=\frac{x^3}{3}$

$$\frac{1}{3}x^3\ln x-\frac{1}{3}\int x^2\,dx=\frac{1}{3}x^3\ln x-\frac{1}{9}x^3+C==\frac{1}{3}x^3\left(\ln x-\frac{1}{3}\right)+C$$

55. $\int x^2\,\text{sen}\,3x\,dx$

$u=x^2 \xrightarrow{\text{derivar}} u'=2x$

$v'=\text{sen}\,3x \xrightarrow{\text{integrar}} v=-\frac{1}{3}\cos 3x$

$$\int x^2\,\text{sen}\,3x\,dx=-\frac{1}{3}x^2\cos 3x+\frac{2}{3}\int x\cos 3x\,dx$$

$u=x \xrightarrow{\text{derivar}} u'=1$

$v'=\cos 3x \xrightarrow{\text{integrar}} v=\frac{1}{3}\text{sen}\,3x$

$$\int x^2\,\text{sen}\,3x\,dx=-\frac{1}{3}x^2\cos 3x+\frac{2}{3}\left(\frac{1}{3}x\,\text{sen}\,3x-\frac{1}{3}\int\text{sen}\,3x\,dx\right)=$$

$$=-\frac{1}{3}x^2\cos 3x+\frac{2}{9}x\,\text{sen}\,3x+\frac{2}{27}\cos 3x+C$$

56. $\int \text{arc sen}\,x\,dx$

$u = \text{arc sen } x \xrightarrow{\text{derivar}} u' = \dfrac{1}{\sqrt{1-x^2}}$

$v' = 1 \xrightarrow{\text{integrar}} v = x$

$$\int \text{arc sen} x\, dx = x \text{ arc sen} x + \int \frac{x}{\sqrt{1-x^2}} dx =$$

$$= x \text{ arc sen} x + \sqrt{1-x^2} + C$$

57. $\displaystyle\int \frac{3^x}{1+3^x} dx$

$3^x = t$

$3^x \ln 3\, dx = dt \qquad dx = \dfrac{dt}{t \cdot \ln 3}$

$$\int \frac{t}{1+t}\,\frac{dt}{t\cdot \ln 3} = \frac{1}{\ln 3}\int \frac{dt}{1+t} = \frac{1}{\ln 3}\ln(1+t) + C =$$

$$= \frac{1}{\ln 3}\ln(1+3^x) + C$$

58. $\displaystyle\int \frac{4e^{3x}}{1+e^{2x}} dx$

$e^x = t$

$e^x dx = dt \qquad dx = \dfrac{dt}{t}$

$$\int \frac{4e^{3x}}{1+e^{2x}} dx = 4\int \frac{t^3}{(1+t^2)t} dt = 4\int \frac{t^2}{1+t^2} dt = 4\int \frac{t^2+1-1}{1+t^2} dt =$$

$$4\left(\int dt - \int \frac{1}{1+t^2} dt\right) = 4(t - \text{arc tg} t) + C =$$

$$= 4(e^x - \text{arc tg } e^x) + C$$