

Ejercicios de inecuaciones

1. $2(x+1) - 3(x-2) < x+6$
2. $\frac{3x+1}{7} - \frac{2-4x}{3} \geq \frac{-5x-4}{14} + \frac{7x}{6}$
3. $6\left(\frac{x+1}{8} - \frac{2x-3}{16}\right) > 3\left(\frac{3}{4}x - \frac{1}{4}\right) - \frac{3}{8}(3x-2)$
4. $\frac{2}{3}\left[x - \left(1 - \frac{x-2}{3}\right)\right] + 1 \leq x$
5. $2 - \left[-2 \cdot (x+1) - \frac{x-3}{2}\right] \leq \frac{2x}{3} - \frac{5x-3}{12} + 3x$
6. $x^2 - 6x + 8 > 0$
7. $x^2 + 2x + 1 > 0$
8. $x^2 + x + 1 > 0$
9. $7x^2 + 21x - 28 < 0$
10. $-x^2 + 4x - 7 < 0$
11. $4x^2 - 16 \geq 0$
12. $4x^2 - 4x + 1 \leq 0$
13. $x^4 + 12x^3 - 64x^2 > 0$
14. $x^4 - 25x^2 + 144 < 0$
15. $x^4 - 16x^2 - 225 \geq 0$
16. $\frac{x-2}{x-4} \geq 0$
17. $\frac{x^2+4}{x^2-4} \geq 0$
18. $\frac{x^2-1}{-x^2+2x-1} \leq 0$
19. $\frac{x^2-1}{x^2-4} \leq 0$
20. $\frac{x+3}{x-2} < 2$
21. $\begin{cases} 2x+3 \geq 1 \\ -x+2 \geq -1 \end{cases}$
22. $\begin{cases} 2x+3 \geq 1 \\ -x+2 < -1 \end{cases}$
23. $\begin{cases} 2x+3 < 1 \\ -x+6 < 3 \end{cases}$
24. $\begin{cases} (x+1) \cdot 10 + x \leq 6(2x+1) \\ 4(x-10) < -6(2-x) - 6x \end{cases}$
25. $\begin{cases} x \geq 4 \\ y \geq 2 \end{cases}$
26. $\begin{cases} x+y \geq 0 \\ 2x-y \geq 0 \end{cases}$
27. $\begin{cases} x+y \geq 0 \\ 2x-y \geq 0 \\ x \leq 6 \end{cases}$

Soluciones

1. $2(x+1) - 3(x-2) < x+6$

$$2x + 2 - 3x + 6 < x + 6$$

$$2x - 3x - x < -2 - 6 + 6$$

$$-2x < -2 \quad x > 1$$



$$(1, \infty)$$

2. $\frac{3x+1}{7} - \frac{2-4x}{3} \geq \frac{-5x-4}{14} + \frac{7x}{6}$

$$m.c.m.(7, 3, 14, 6) = 42$$

$$6(3x+1) - 14(2-4x) \geq 3(-5x-4) + 49x$$

$$18x + 6 - 28 + 56x \geq -15x - 12 + 49x$$

$$18x + 56x + 15x - 49x \geq -12 - 6 + 28$$

$$40x \geq 10$$

$$4x \geq 1$$

$$x \geq \frac{1}{4}$$



$$\left[\frac{1}{4}, \infty\right)$$

3. $6\left(\frac{x+1}{8} - \frac{2x-3}{16}\right) > 3\left(\frac{3}{4}x - \frac{1}{4}\right) - \frac{3}{8}(3x-2)$

$$\frac{6(x+1)}{8} - \frac{6(2x-3)}{16} > \frac{9}{4}x - \frac{3}{4} - \frac{9}{8}x + \frac{6}{8}$$

$$m.c.m.(8, 16, 4) = 16$$

$$\cancel{12x} + 12 - \cancel{12x} + 18 > 36x - \cancel{12} - 18x + \cancel{12}$$

$$12 + 18 > 36x - 18x$$

$$18x < 30 \quad 3x < 5 \quad x < \frac{5}{3}$$

$$\left(-\infty, \frac{5}{3}\right)$$

$$4. \frac{2}{3} \left[x - \left(1 - \frac{x-2}{3} \right) \right] + 1 \leq x$$

$$\frac{2}{3} \left(x - 1 + \frac{x-2}{3} \right) + 1 \leq x$$

$$\frac{2}{3}x - \frac{2}{3} + \frac{2x-4}{9} + 1 \leq x$$

$$6x - 6 + 2x - 4 + 9 \leq 9x$$

$$-x \leq 1 \quad x \geq -1$$



$$[-1, \infty)$$

$$5. 2 - \left[-2 \cdot (x+1) - \frac{x-3}{2} \right] \leq \frac{2x}{3} - \frac{5x-3}{12} + 3x$$

$$2 - \left(-2x - 2 - \frac{x-3}{2} \right) \leq \frac{2x}{3} - \frac{5x-3}{12} + 3x$$

$$2 + 2x + 2 + \frac{x-3}{2} \leq \frac{2x}{3} - \frac{5x-3}{12} + 3x$$

$$24 + 24x + 24 + 6 \cdot (x-3) \leq 8x - (5x-3) + 36x$$

$$24 + 24x + 24 + 6x - 18 \leq 8x - 5x + 3 + 36x$$

$$24x + 6x - 8x + 5x - 36x \leq 3 - 24 - 24 + 18$$

$$-9x \leq -27$$

$$9x \geq 27$$

$$x \geq 3$$

$$6. x^2 - 6x + 8 > 0$$

$$x^2 - 6x + 8 = 0$$

$$x = \frac{6 \pm \sqrt{6^2 - 4 \cdot 8}}{2} = \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm 2}{2} = \begin{matrix} \nearrow x_1 = \frac{8}{2} = 4 \\ \searrow x_2 = \frac{4}{2} = 2 \end{matrix}$$

$$P(0) = 0^2 - 6 \cdot 0 + 8 > 0$$

$$P(3) = 3^2 - 6 \cdot 3 + 8 = 17 - 18 < 0$$

$$P(5) = 5^2 - 6 \cdot 5 + 8 = 33 - 30 > 0$$



$$S = (-\infty, 2) \cup (4, \infty)$$

7. $x^2 + 2x + 1 > 0$

$$x^2 + 2x + 1 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4}}{2} = \frac{-2 \pm 0}{2} = -1$$

$$S = \mathbb{R}$$

8. $x^2 + x + 1 > 0$

$$x^2 + x + 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$P(0) = 0 + 0 + 1 > 0$$

El signo obtenido coincide con el de la desigualdad, la solución es $\mathbb{R}$.

9. $7x^2 + 21x - 28 < 0$

$$x^2 + 3x - 4 < 0$$

$$x^2 + 3x - 4 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm 5}{2} = \begin{matrix} \nearrow x_1 = 1 \\ \searrow x_2 = -4 \end{matrix}$$

$$P(-6) = (-6)^2 + 3 \cdot (-6) - 4 > 0$$

$$P(0) = 0^2 + 3 \cdot 0 - 4 < 0$$

$$P(3) = 3^2 + 3 \cdot 3 - 4 > 0$$



$$(-4, 1)$$

10. $-x^2 + 4x - 7 < 0$

$$x^2 - 4x + 7 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 28}}{2} = \frac{4 \pm \sqrt{-12}}{2} \notin \mathbb{R}$$

$$P(0) = -0^2 + 4 \cdot 0 - 7 < 0$$

$$S = \mathbb{R}$$

11. $4x^2 - 16 \geq 0$

$$4x^2 = 16$$

$$x^2 = 4 \quad x = \pm \sqrt{4} \quad \begin{matrix} \nearrow x_1 = 2 \\ \searrow x_2 = -2 \end{matrix}$$



$$P(-3) = 4 \cdot (-3)^2 - 16 > 0$$

$$P(0) = 4 \cdot 0^2 - 16 < 0$$

$$P(3) = 4 \cdot 3^2 - 16 > 0$$



$$(-\infty, -2] \cup [2, +\infty)$$

12. $4x^2 - 4x + 1 \leq 0$

$$4x^2 - 4x + 1 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{4}{8} = \frac{1}{2}$$

$$\left(x - \frac{1}{2}\right)^2 \leq 0 \quad x = \frac{1}{2}$$

13. $x^4 + 12x^3 - 64x^2 > 0$

$$x^2(x^2 + 12x - 64) > 0$$

$$x^2 + 12x - 64 = 0$$

$$x = \frac{-12 \pm \sqrt{144 + 256}}{2} = \frac{-12 \pm 20}{2} = \begin{matrix} \nearrow x_2 = 4 \\ \searrow x_3 = -16 \end{matrix}$$



$$P(-17) = (-17)^2 + 12 \cdot 17 - 64 > 0$$

$$P(0) = 0^2 + 12 \cdot 0 - 64 < 0$$

$$P(5) = 5^2 + 12 \cdot 5 - 64 > 0$$



$$(-\infty, -16] \cup [4, \infty)$$

14. $x^4 - 25x^2 + 144 < 0$

$$x^4 - 25x^2 + 144 = 0$$

$$x^2 = t$$

$$t^2 - 25t + 144 = 0$$

$$t = \frac{25 \pm \sqrt{625 - 576}}{2} = \frac{25 \pm 7}{2} = \begin{cases} t_1 = 16 \\ t_2 = 9 \end{cases}$$

$$x^2 = 16 \quad x = \pm\sqrt{16} \quad \begin{cases} x_1 = 4 \\ x_2 = -4 \end{cases}$$

$$x^2 = 9 \quad x = \pm\sqrt{9} \quad \begin{cases} x_1 = 3 \\ x_2 = -3 \end{cases}$$



$$(-4, -3) \cup (-3, 3) \cup (3, 4).$$

15. $x^4 - 16x^2 - 225 \geq 0$

$$x^4 - 16x^2 - 225 = 0$$

$$x^2 = t$$

$$t^2 - 16t - 225 = 0$$

$$t = \frac{16 \pm \sqrt{256 + 900}}{2} = \frac{16 \pm 34}{2} = \begin{cases} t_1 = 25 \\ t_2 = -9 \end{cases}$$

$$x^2 = 25 \quad x = \pm\sqrt{25} \quad \begin{cases} x_1 = 5 \\ x_2 = -5 \end{cases}$$

$$x^2 = -9 \quad x = \pm\sqrt{-9} \notin \mathbb{R}$$

$$(x^2 - 25) \cdot (x^2 + 9) \geq 0$$



$$(-\infty, -5] \cup [5, +\infty)$$

16. $\frac{x-2}{x-4} \geq 0$

$$x - 2 = 0 \quad x = 2$$

$$x - 4 = 0 \quad x = 4$$



$$x = 0 \quad \frac{0-2}{0-4} > 0$$

$$x = 3 \quad \frac{3-2}{3-4} < 0$$

$$x = 5 \quad \frac{5-2}{5-4} > 0$$



$$S = (-\infty, 2] \cup (4, \infty)$$

17. $\frac{x^2 + 4}{x^2 - 4} \geq 0$

$$x^2 + 4 = 0$$

$$x = \pm\sqrt{-4} \notin \mathbb{R}$$

$$x^2 - 4 = 0$$

$$x = \pm\sqrt{4} \begin{matrix} \nearrow x_1 = 2 \\ \searrow x_2 = -2 \end{matrix}$$



$$(-\infty, -2) \cup (2, +\infty)$$

18. $\frac{x^2 - 1}{-x^2 + 2x - 1} \leq 0$

$$x^2 - 1 = 0$$

$$x = \pm 1$$

$$-x^2 + 2x - 1 = 0$$

$$x^2 - 2x + 1 = 0$$

$$x = 1$$



$$(-\infty, -1] \cup (1, +\infty)$$

19. $\frac{x^2 - 1}{x^2 - 4} \leq 0$

$$x^2 - 1 = 0$$

$$x = \pm 1$$

$$x^2 - 4 = 0$$

$$x = \pm 2$$



$$[-2, -1] \cup (1, 2)$$

20. $\frac{x+3}{x-2} < 2$

$$\frac{x+3}{x-2} - 2 < 0$$

$$\frac{x+3-2(x-2)}{x-2} < 0$$

$$\frac{-x+7}{x-2} < 0$$

$$-x+7=0 \quad x=7$$

$$x-2=0 \quad x=2$$

$$x=0 \quad \frac{0+7}{0-2} < 0$$

$$x=3 \quad \frac{3+7}{3-2} > 0$$

$$x=8 \quad \frac{-8+7}{8-2} < 0$$



$$S = (-\infty, 2) \cup (7, \infty)$$

21. $\begin{cases} 2x+3 \geq 1 \\ -x+2 \geq -1 \end{cases}$

$$2x+3 \geq 1$$

$$2x \geq 1-3$$

$$2x \geq -2$$

$$x \geq -1$$

$$-x+2 \geq -1$$

$$-x \geq -1-2$$

$$-x \geq -3$$

$$x \leq 3$$



$$[-1, 3]$$

22. $\begin{cases} 2x+3 \geq 1 \\ -x+2 < -1 \end{cases}$

$$2x+3 \geq 1$$

$$2x \geq -2$$

$$x \geq -1$$

$$-x+2 < -1$$

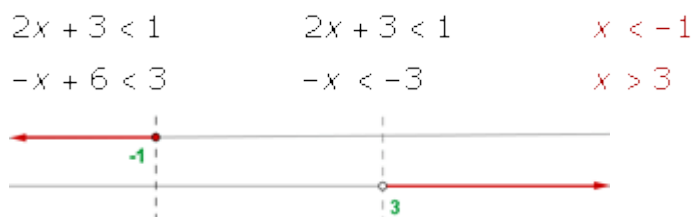
$$-x < -3$$

$$x > 3$$



$$(3, \infty)$$

23.
$$\begin{cases} 2x + 3 < 1 \\ -x + 6 < 3 \end{cases}$$



No tiene solución.

24.
$$\begin{cases} (x+1) \cdot 10 + x \leq 6(2x+1) \\ 4(x-10) < -6(2-x) - 6x \end{cases}$$

1ª inecuación:

$$(x+1) \cdot 10 + x \leq 6(2x+1)$$

$$10x + 10 + x \leq 12x + 6$$

$$10x + x - 12x \leq 6 - 10$$

$$-x \leq -4 \quad x \geq 4$$

2ª inecuación:

$$4(x-10) < -6(2-x) - 6x$$

$$4x - 40 < -12 + 6x - 6x$$

$$4x - 6x + 6x < -12 + 40$$

$$4x < 28 \quad x < 7$$

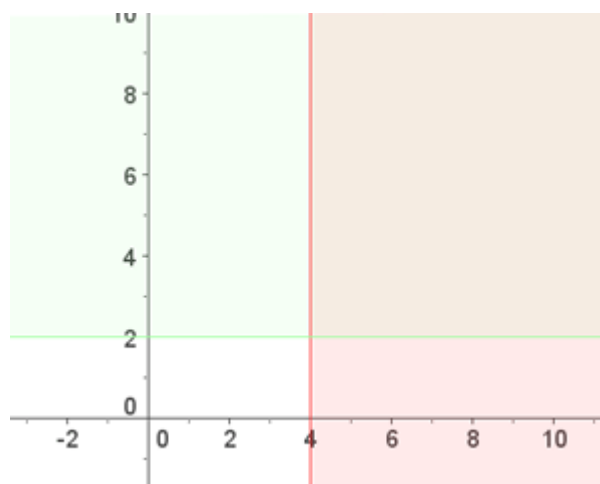


[4, 7)

25.
$$\begin{cases} x \geq 4 \\ y \geq 2 \end{cases}$$

$$x = 4$$

$$y = 2$$



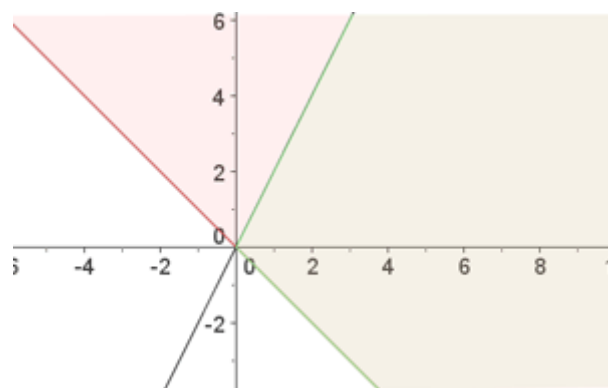
26.
$$\begin{cases} x + y \geq 0 \\ 2x - y \geq 0 \end{cases}$$

$x + y = 0$ (0, 0) (1, -1)

$2 + 2 \geq 0$

$2x - y = 0$ (0, 0) (1, 2)

$2 \cdot 2 - 2 \geq 0$



27.
$$\begin{cases} x + y \geq 0 \\ 2x - y \geq 0 \\ x \leq 6 \end{cases}$$

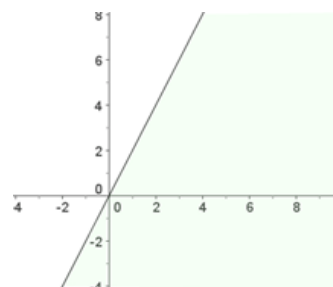
$x + y = 0$ (0, 0) (1, -1)

$2 + 2 \geq 0$



$2x - y = 0$ (0, 0) (1, 2)

$2 \cdot 2 - 2 \geq 0$



$2 \leq 6$

