Ejercicios de logaritmos

1. Calcular por la **definición de logaritmo** el valor de y.

1. $\log_{\frac{1}{2}} 0.25 = y$
2. $\log_{\sqrt{5}} 125 = y$
3. $\log 0.001 = y$
4. $\log_{\sqrt{3}} \sqrt[5]{\frac{1}{81}} = y$

2. Calcula el valor de **x** aplicando la **definición de logaritmo**:

1. $\log_2 32 = x$
2. $\log_9 \frac{1}{3} = x$
3. $\log_{\frac{1}{2}} 0.25 = x$
4. $\log_9 \sqrt[4]{3} = x$
5. $\log_{\sqrt{2}} \frac{1}{4} = x$
6. $\log_x 81 = -4$
7. $\log_2 x^3 = 6$

3. Conociendo que log 2 = 0.3010, **calcula** los siguientes **logaritmos decimales**.

1. $\log 0.02$
2. $\log \sqrt[4]{8}$
3. $\log 5$
4. $\log 0.0625$

4. Calcular los **logaritmos** de de las expresiones que se indican:

1. $\log_2 \frac{a^2 - b^2}{a \cdot b} =$
2. $\log \sqrt{2\sqrt{2\sqrt{2}}}$

Soluciones

1. Calcular por la **definición de logaritmo** el valor de y.

1. $\log_{\frac{1}{2}} 0.25 = y$ $\quad \left(\frac{1}{2}\right)^y = 0.25$ $\quad \left(\frac{1}{2}\right)^y = \left(\frac{1}{2}\right)^2$ $\quad y = 2$

2. $\log_{\sqrt{5}} 125 = y$ $\quad \sqrt{5}^y = 125$ $\quad 5^{\frac{1}{2}y} = 5^3$ $\quad y = 6$

3. $\log 0.001 = y$ $\quad 10^y = 0.001$ $\quad 10^y = 10^{-3}$ $\quad y = -3$

4. $\log_{\sqrt{3}} \sqrt[5]{\frac{1}{81}} = y$ $\quad \sqrt{3}^y = \sqrt[5]{\frac{1}{81}}$ $\quad 3^{\frac{1}{2}y} = 3^{-\frac{4}{5}}$ $\quad y = -\frac{8}{5}$

2. Calcula el valor de **x** aplicando la **definición de logaritmo**:

1. $\log_2 32 = x$ $\quad 2^x = 32$ $\quad 2^x = 2^5$ $\quad x = 5$

2. $\log_9 \frac{1}{3} = x$ $\quad (9)^x = \frac{1}{3}$ $\quad 3^{2x} = 3^{-1}$ $\quad x = -\frac{1}{2}$

3. $\log_{\frac{1}{2}} 0.25 = x$ $\quad \left(\frac{1}{2}\right)^x = \frac{25}{100}$ $\quad \left(\frac{1}{2}\right)^x = \left(\frac{1}{4}\right)$

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^2$ $\quad x = 2$

4. $\log_9 \sqrt[4]{3} = x$ $\quad (9)^x = \sqrt[4]{3}$ $\quad 3^{2x} = 3^{\frac{1}{4}}$ $\quad x = \frac{1}{8}$

5. $\log_{\sqrt{2}} \frac{1}{4} = x$ $\quad \left(\sqrt{2}\right)^x = \frac{1}{4}$ $\quad 2^{\frac{1}{2}x} = 2^{-2}$ $\quad x = -4$

6. $\log_x 81 = -4$ $\quad x^{-4} = 81$ $\quad x^4 = \frac{1}{81}$ $\quad x = \frac{1}{3}$

7. $\log_2 x^3 = 6$ $\quad x^3 = 2^6$ $\quad x = 4$

3. Conociendo que log 2 = 0.3010, **calcula** los siguientes **logaritmos decimales**.

1. $\log 0.02$ $\quad \log\left(\frac{2}{100}\right) = \log 2 - \log 10^2 = \log 2 - 2 = 0.3010 - 2 = -1.699$

2. $\log \sqrt[4]{8}$

$$\log \sqrt[4]{2^3} = \frac{3}{4}\log 2 = \frac{3}{4}\cdot 0.3010 = 0.2257$$

3. $\log 5$

$$\log\left(\frac{10}{2}\right) = \log 10 - \log 2 = 1 - 0.3010 = 0.699$$

4. $\log 0.0625$

$$\log\left(\frac{625}{10000}\right) = \log\left(\frac{5^4}{2^4\cdot 5^4}\right) = \log\left(\frac{1}{2^4}\right) =$$

$$\log 1 - \log 2^4 = 0 - 4\log 2 = -1.2040$$

4. Calcular los **logaritmos** de de las expresiones que se indican:

1. $\log_2 \dfrac{a^2-b^2}{a\cdot b} =$

$$= \log_2 \frac{(a+b)\cdot(a-b)}{a\cdot b} =$$

$$= \log_2\left[(a+b)\cdot(a+b)\right] - \log_2(a\cdot b) =$$

$$= \log_2(a+b) + \log_2(a-b) - (\log_2 a + \log_2 b) =$$

$$= \log_2(a+b) + \log_2(a-b) - \log_2 a - \log_2 b$$

2. $\log\ 2\sqrt{2\sqrt{2\sqrt{2}}}$

$$\log\ 2\sqrt{2\sqrt{2\sqrt{2}}} = \log\ 2 + \log\ \sqrt{2\sqrt{2\sqrt{2}}} =$$

$$= \log\ 2 + \frac{1}{2}\log\left(2\sqrt{2\sqrt{2}}\right) = \log\ 2 + \frac{1}{2}\left[\log\ 2 + \frac{1}{2}\log\left(2\sqrt{2}\right)\right] =$$

$$= \log\ 2 + \frac{1}{2}\left[\log\ 2 + \frac{1}{2}\left(\log\ 2 + \log\sqrt{2}\right)\right] =$$

$$= \log\ 2 + \frac{1}{2}\left[\log\ 2 + \frac{1}{2}\left(\log\ 2 + \frac{1}{2}\log 2\right)\right] =$$

$$= \log\ 2 + \frac{1}{2}\log\ 2 + \frac{1}{4}\log\ 2 + \frac{1}{8}\log 2 = \frac{15}{8}\log\ 2$$